Danish Research Institute for Economic Analysis and Modelling

A short note on Perceived Risk

Peter Stephensen

Note
9 September 2022

www.dreamgruppen.dk

A short note on Perceived Risk

Peter Stephensen*

September 7, 2022

In standard porfolio theory, mean-variance analysis is used to assamble a portfolio of assets such that the expected return is maximized for a given level of risk (Markowitz, 1952)[1]. Here we give a behavioral version of the same story. The investors risk preferences is described by a *Percieved-Risk-function*. According to this function, more risk is associated with investing more, and investing un-deversified. It is demonstrated that if this function is a *Constant Elasticity of Transformation* function (CET function), the fortfolio problem has a simple analytical solution.

An investor is investing a nominal sum S in n different assets. The j'th assets has returns ρ_j. We wish to describe the investor's portfolio. The central idea is that the investor measures her *perceived risk*, R, with a CET-function:

$$R = \left[\sum_i \mu_i^{\frac{1}{E}} s_i^{\frac{E-1}{E}} \right]^{\frac{E}{E-1}}, E < 0$$

where s_i is the amount invested in asset i. The more that is invested, the higher the risk (scale effect), and the less balanced the portfolio is, the higher the risk (deversification effect). The latter is ensured through the assumption that $E < 0$, or in other words that we use a CET-function.

We now assume, that for a given risk-level R, the investor maximizes the total returns $\sum_i \rho_i s_i$. This results in the CET-system:

$$s_i = \mu_i \left(\frac{\rho_i}{\rho^R} \right)^{-E} R, i = 1, .., n \tag{0.1}$$

$$\rho^R R = \sum_i \rho_i s_i$$

The investor has a total sum S, which should be split across different assets. The investor therefore chooses the risk-level R, which ensures that:

*Danish Research Institute for Economic Analysis and Modeling, DREAM, Landgreven 4, 7th floor DK-1301 Copenhagen K, E-mail: psp@dreamgruppen.dk [v0.2].

[1]Harry Markowitz (1952) *Portfolio Selection*. The Journal of Finance, Vol. 7, No. 1. (Mar., 1952), pp. 77-91.

$$\sum_i s_i = S$$

It follows from (0.1) that

$$\sum_i s_i = R \sum_i \mu_i \left(\frac{\rho_i}{\rho^R} \right)^{-E} = S$$

such that

$$R = \frac{S}{\sum_i \mu_i \left(\frac{\rho_i}{\rho^R} \right)^{-E}}$$

Inserting this in (0.1) implies that:

$$s_i = \frac{\mu_i \rho_i^{-E}}{\sum_j \mu_j \rho_j^{-E}} S, \; i = 1, .., n \tag{0.2}$$

Notice that the individual assets, s_i, will always sum to the total sum (we have a portfolio theory) and that if the return on an asset i increases, so too will its share of the portfolio (since $E < 0$). Notice further that the abstract concepts R and ρ^R have disappeared from the solution.

The average return can be calculated as:

$$\rho \equiv \frac{\sum_i \rho_i s_i}{S}.$$

The average return is maximized by the investor, as she is maximizing the numerator for given value of the denominator. Inserting (0.2), we find that:

$$\rho = \sum_i \frac{\mu_i \rho_i^{-E}}{\sum_j \mu_j \rho_j^{-E}} \rho_i,$$

such that the average return is a well-defined price index of the individual returns.