DREAM

Danish Research institute for
Economic Analysis and Modelling

MAKRO

Documentation af MAKRO version 18AUG

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Documentation
18 September 2018

www.dreamgruppen.dk

MAKRO

September 18, 2018

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Documentation of version MAKRO 18AUG (Preliminary draft)

Contents

1 Introduction

This documentation is a description of the code as it is in the model version MAKRO 18AUG. It is mainly written for the model users in the ministries to give them a better understanding of the background for the code. The model solves, has a coherent structure based on optimizing agents and is calibrated on Danish data. It has, however, not yet been matched to have empirically aligned impulse-responses. The model represents work in progress, and the modeling of different areas described in this document does not necessarily reflect the modeling, that will ultimately be part of MAKRO at the end of the development period. Thus it does not necessarily represent the modeling or approaches used in the current (or will be used in future) short and long term projections in the Ministry of Finance. Many of the specific details in the model will change during the next few years: A more detailed labor market and explicit housing market - just to name a few. The documentation should, however, give an idea about how MAKRO will look like once it is finished. Not only the model, but also the documentation itself is work in progress as we try to update it as we implement new model expansions. As a consequence the editorial quality is not that of a published journal article or a documentation for a finished model. We in advance apologize for typos and smaller parts of the text not fully in line with the newest expansions.

MAKRO is a large model. One of its main purposes is determining the government budget balance, the structural budget balance, and the effect from shocks/policy on the budget balance. The structural budget balance is the budget balance adjusted for business cycle movements. This is calculated taking the actual budget balance and adjusting for the output gap and a few other temporary conditions. The government budget balance consists of government income net government expenditures. The expenditures consist mainly of government consumption and income transfers. Both government consumption and income transfers depend on demographics. They also depend on the wage level - as both public wages and income transfers are regulated according to the overall wage level. Income transfers also depends on the unemployment level. In this version of the model, government consumption is exogenous. Government income consists mainly of taxes and duties. The main tax component is personal income taxes and depends on both unemployment and the wage level. Corporate taxes depend on the activity and firm earnings. Duties depend not only on the aggregated demand, but also on the composition of the demand. The government budget is described in more detail in chapter 2.

Two of the most important components of the budget balance is employment and wages. Both depend on demand. Demand consists of material inputs, private consumption, government consumption, investments, and exports. Material inputs depend on and are used in firm production. Private consumption depends on real discounted household income and wealth as well as demographics. Investments depend on the optimal capital stock, which depends on the level of production, user cost, and expectations. The user cost is sensitive to changes in the interest rate. Exports depends on the foreign demand, which is partly determined by the economic activity abroad and the relative prices of foreign and domestic goods and services.

Firm production is modeled in detail - where firms in each sector optimize their value given their production function and demand curve. Firms use capital, labor, and materials in production. Quadratic adjustment costs to capital make it expensive to make large adjustments to the capital stock. In this way the investment decisions of the firms depend on not only the current interest rate and demand, but also on the expected future demand and interest rate. In the short run, it is expensive to meet increased demand by large increases in the capital stock and labor demand must compensate. The firm is described in more detail in chapter 3.

Public production is given as the sum of exogenous inputs to public production, public sales to private consumption and direct public investments in R&D. Public production is calculated by the input method and there is no productivity growth implying a higher growth in the deflator than in private sectors. The public production is described in more detail in chapter 4.

Firms set prices to maximize the value of the firm. Given their demand functions they want to set a higher price when their production costs increase. Nominal rigidities formulated with a Calvo contract makes this adjustment rigid. The price setting is described in more detail in chapter 5.

Employment is given by the demand for labor. Labor market participation is exogenous in this version of the model and unemployment is residually given. A Phillips curve makes wages increase more when unemployment is below its structural level and less when it is above. Both labor market participation, structural unemployment, and productivity is age dependent. Demographics are exogenously given. In calculating structural output the wage is set such that the actual unemployment equals its structural level. The labor market is described in more detail in chapter 6.

Consumption depends on expected discounted real income and wealth as well as demographics. It is modeled with an overlapping generations model with one cohort for each year. A fraction of the consumers are hand to mouth and spend their entire income each period. This is necessary to get a consumption response to income changes that match the data. The other consumers maximize utility given their wealth and the expected discounted value of income. This gives them a nice smooth consumption over their lifespan. So demographics will have an effect on the aggregate consumption even for a given aggregate income and wealth. Habit formation gives rigidity in the consumption to match the data. The households are described in more detail in chapter 7.

Households consume different goods and services. Aggregated private consumption is divided into consumption groups assuming all cohorts have identically nested CES functions. More details on consumption decomposition are given in chapter 8.

Exports are modeled using an Armington approach and depends on the expected import growth from our trade partners and our price competitiveness. Our trade partners expected import growth is exogenously given. Our price competitiveness is given as the export prices relative to export competing foreign prices. The foreign sector is described in more detail in chapter 9.

The demand for materials, private consumption, government consumption, investments, and exports must be met by supply from domestic and foreign sectors. This is taken care of by the Input-/Output system. Each demand component has a fixed input structure from the different sectors. This input can come from either domestic production or imports. The demand for a given sector's domestic production is given by the sum over all demand components for domestic deliveries from this sector. Similarly, imports from a given sector is the sum of imports for all demand components from this sector. Each demand component in each sector has a substitution between imports and domestic production. If domestic prices increase relative to import prices then imports increase. The most dis-aggregated prices are the sector specific prices after taxes. Price aggregates, including the prices for all demand components, are CES price indices. Imports and the Input-/Output system is described in detail in chapter 10.

In the chapters mentioned above there are short descriptions on how the parameters from that part of the model are calibrated. Chapter 11 gives an overview of the calibration process as a whole and describes the methods used.

In the next sub-section follows a discussion on structural levels in the model. The three last subsections try to tie the documentation to the code. Readers, who have never seen the code and has no ambition to look in the code, can skip the last three subsections and go directly to the other chapters.

1.1 Modules in the code

The model is coded in GAMS. It is divided into different modules. The modules are covered in the coming chapters. The chapter numbers most relevant to the module are indicated in parentheses. The modules are: Government (ch. 2), Firms (ch. 3), Production (Ch.3&4), Pricing(ch. 5), Labor market (ch. 6), Consumers (Ch. 7&8), Foreign sector (Ch. 9), IO (Ch. 10), Taxes (Ch. 10), and Aggregates (Ch. 10). The modules can be solved separately, but require further important inputs from other modules and provide important outputs to other files. Figure 1.1 is a diagram showing the inputs and outputs from the different files (NB: This diagram is not updated to 18AUG.).

To give an example of the structure: Government consumption and budget balance is calculated in the government file. As input this file needs the wage and number of people unemployed from the labor market module as well as duties, VAT, production taxes, and corporate taxes from the Taxes module and GDP from the Aggregates module.

1.2 Notation

Notation in this document is consistent with the code, but it is not identical. In the code all quantities have prefix q, all prices have prefix p and all values have prefix v. In this text we write quantities without prefix and prices as P with the rest of the name as a superscript to ease notation. In the documentation, sets are noted as subscripts, but the tot subscript is suppressed when writing aggregates. When possible, values are written as their price times their quantity. If no quantity exists, values are written without prefix. Share parameters are written as μ instead of s and elasticities are written as σinstead of e. Examples:

$$qY[s,t] = Y_{s,t}$$

$$pY[s,t] = P^Y_{s,t}$$

$$vY[s,t] = P^Y_{s,t} Y_{s,t}$$

$$qK['tot', s, t] = K_{s,t}$$

$$qK['tot', 'tot', t] = K_t$$

$$vKbook[k,s,t] = K^{book}_{k,s,t}$$

$$qI['iM', t] = I_{iM,t}$$

$$sL[s,t] = \mu^{LKL}_{s,t}$$

$$eKL[s] = \sigma^{KL}_s$$

In the literature Y_t is in different contexts used for production, GDP and gross value added. In ADAM it is used for GDP. Here we follow DREAM and use it for production. GDP is called GDP_t and gross value added is called $GrossValAdd_t$.

The most important sets are time, t=2000,...,2200, age , a=16,...,100, and sector, s=sp,pub=agr,con,ene,ext,hou,man,sea,ser,pub where sp is the private sectors. The demand components have sets tied to the decomposed demand. Material input, $R_{r,t}$, are demanded for each sector s, but we save the suffix s for the delivering sectors and instead use r=s for demand from sectors. Private consumption, $C_{c,t}$, is decomposed into consumption of cars, energy, (other) goods, housing, services and tourism (c=cCar,cEne,cGoo,cHou,cSer,cTou). Government consumption, $G_{g,t}$, only has one component g=g, but is coded to be able to include more. The three types of investment in each sector, $I_{i,s,t}$, has the suffix i=iM,iB,Invt indicating investments in machinery, buildings and inventories. Since there is no capital stock for inventories we here use $K_{k,s,t}$ with k=iM,iB. Exports, $X_{x,t}$, are demand for goods, services and tourism x = xEne,xGoo,xSea,xSer,xTou. In the code many variables are defined over sets including a total called 'tot'. These sets have the same name, but with an underscore for example k_=k,tot=iM,iB,tot.

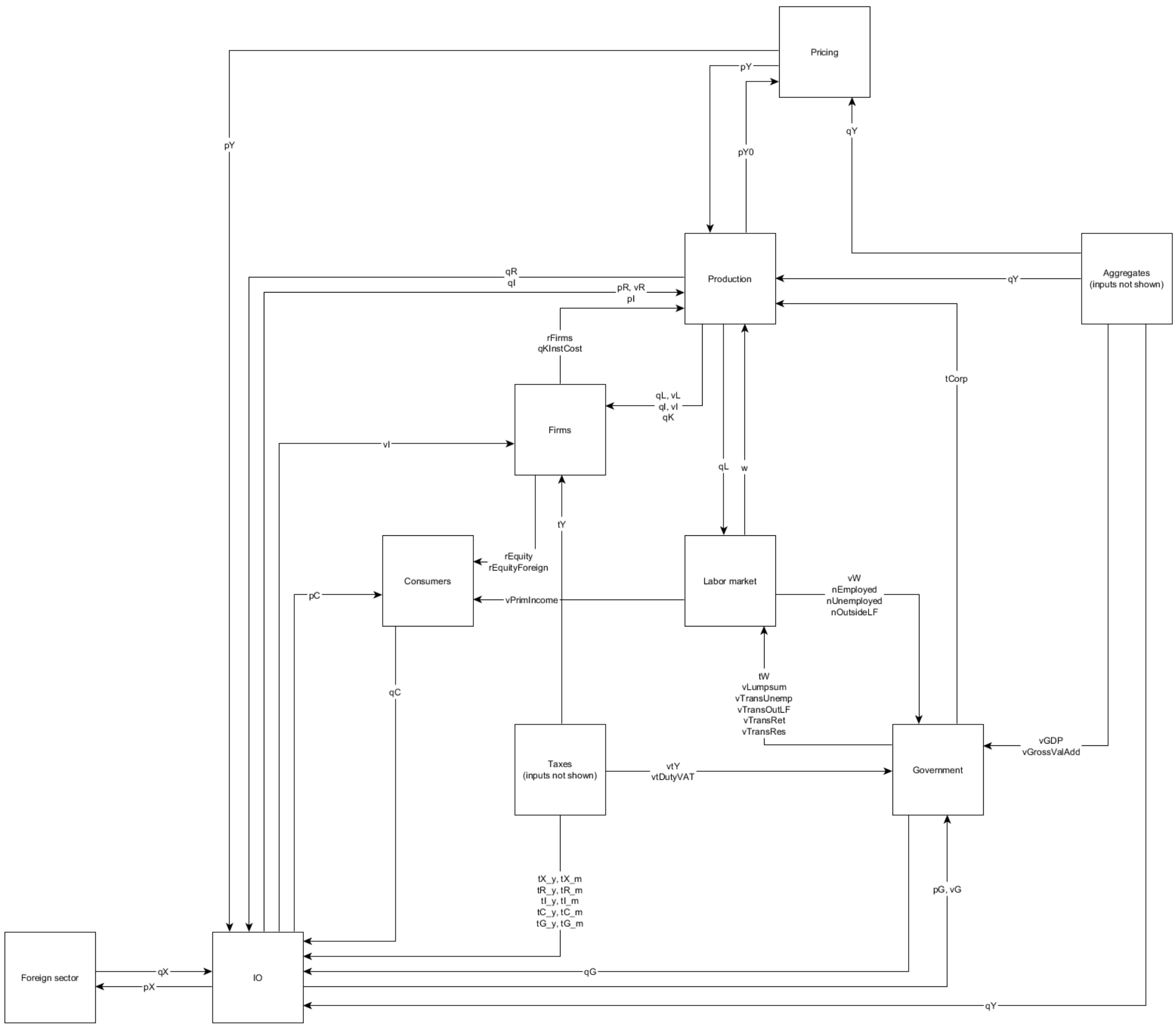


Figure 1.1.1: The figure shows endogenous variables that are part of more than one module. Arrows point from the module in which a variable is defined to other modules that use the variable. The variables displayed can also be seen as the endogenous inputs and outputs to and from modules.The aggregates and taxes modules have inputs from almost every other module. The tax module calculates revenues from taxes on most supply and demand components. The aggregates module contains market equilibrium conditions and aggregate national accounting identities.

2 The government sector

The main focus of this section is the government budget balance, the structural budget balance and fiscal sustainability. The government sector includes a detailed modeling of public revenues and expenditures. Some variables lack a satisfying modeling, which mainly is due to limitations from the level of development in the other sections. The structural budget balance and its components are modeled and the fiscal sustainability indicator is introduced.

The appendix for government variables contains a detailed overview table of all public revenues and expenditures; their name, value, how they are corrected regarding structural level, and which ADAM variable they are equal to.

2.1 Overview

The government sector is focused around the budget balance:

$$GovBalance_t = GovPrimBalance_t + GovNetInterest_t$$

The primary budget is given by:

$$GovPrimBalance_t = GovRev_t - GovExp_t$$

The net interest payments are given by:

$$GovNetInterest_t = GovInterest_t^{Assets} - GovInterest_t^{Debt}$$

The modeling of government interests, assets, debt and wealth accumulation is described in 2.2.

Government revenue is given by:

$$GovRev_t = T_t^{Direct} + T_t^{Indirect} + GovRev_t^{Rest}$$

The main part of T_t^{Direct} is general income taxation, but it also includes corporate taxation, taxation on housing and more. The components in T_t^{Direct} are described in details in 2.3.1 and covers many aspects of the Danish income tax system. The indirect taxes, $T_t^{Indirect}$, are mainly given by duties, VAT and production taxes, and are described in details in 2.3.3.
$GovRev_t^{Rest}$ includes other public revenues and is described in 2.3.4. Revenues from $GovRev_t^{Rest}$ is around 10 % of total revenues whereas T_t^{Direct} is the largest contributor with around 60 % of all public revenues.

The government expenditures are given by:

$$GovExp_t = G_t + Trans_t + I_t^{Gov} + GovSub_t + GovExp_t^{Rest}$$

The modeling of the components in government expenditures are described in 2.4.

2.2 The government's net wealth and interests

As described earlier net interest payments from the government is simply determined as interests on assets minus interests on debt. Those two are determined as follows:

$$GovInterest_t^{Assets} = i_t^{GovAssets} GovAssets_{t-1}$$

$$GovInterest_t^{Debt} = i_t^{GovDebt} GovDebt_{t-1}$$

The two interest rates are calibrated in calibration years so as the data for $GovInterest_t^{Assets}$, $GovAssets_{t-1}$, $GovInterest_t^{Debt}$ and $GovDebt_{t-1}$ fits the equations above. In the long run the interest rates will be equal to the risk free interest rate.
The accumulation of the government's net wealth follows:

$$GovWealth_t = GovWealth_{t-1} + GovBalance_t + GovReval_t$$

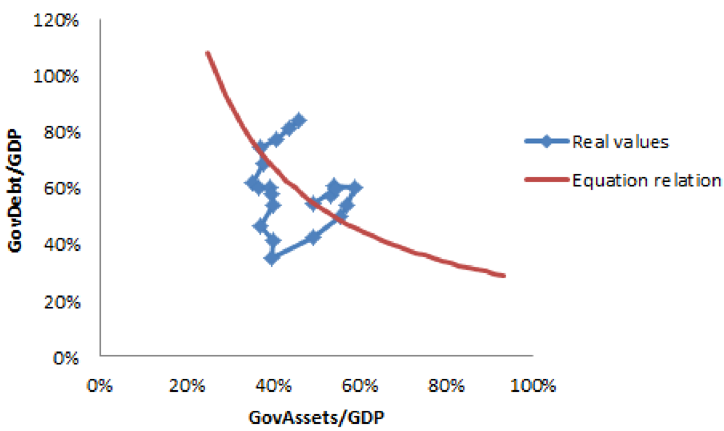
Whereas revaluations, $GovReval_t$, is calibrated in calibrations years so as the data for $GovWealth_t$, $GovWealth_{t-1}$ and $GovBalance_t$ fits the equation above. Revaluations is set to zero in years after the calibration period.
The government's net wealth follows:

$$GovWealth_t = GovAssets_t - GovDebt_t$$

Often government assets is set exogenous, whereas government debt is determined by the equation above. This approach leads to the possibility, that government debt can be negative. To avoid this the following relation between assets and debt is introduced:

$$\frac{GovDebt_t}{GDP_t} = ADparameter \cdot \frac{1}{GovAssets_t/GDP_t}$$

$ADparameter$ is just a calibrated parameter so the equation holds and $GovAssets_t$ and $GovDebt_t$ fits real data. In the figure below one can see how $GovAssets_t$ and $GovDebt_t$ change when $GovWealth_t$ changes. This reaction function is not a result of deep thoughts, empirical work and literature studies. It is introduced to secure that $GovAssets_t$ and $GovDebt_t$ remains positive as it has the following nice properties: $GovAssets_t > 0$,$GovDebt_t > 0$, $\frac{\partial GovDebt_t}{\partial GovWealth_t} < 0$ and $\frac{\partial GovAssets_t}{\partial GovWealth_t} > 0$.



2.3 Public revenues

As previously described, total public revenue is the sum of direct taxation, indirect taxation and other government revenues. In the following 3 subsections the modeling of the revenues are carefully described. Especially the modeling of direct taxation is very extensive. However, the modeling of taxation is not crucial for the general economic mechanisms in the model.

At notation; the tax rates are called t, while the revenues are called T both with superscripts. For example, the sector specific corporate tax rate is called $t_{s,t}^{Corp}$ and the total revenue is called T_t^{Corp}. (In the code they are called tCorp(s,t) and vtCorp('tot',t)).

2.3.1 Direct taxation

Direct taxation is modelled relatively close to the real Danish income tax law[1] with only a few shortcuts. This is done as the revenue from direct taxation is of large significance and because major economical and demographical movements may affect the tax burden so the relationship between direct taxation and the total income level is not constant. Finally, a specific modeling of the tax system makes it easier to make economic-political analyses inside the model frame. The modeling and definitions of income terms and allowances are described in the next subsection.

The direct taxation is mainly given by the income taxes T_t^{Income}. Besides income taxes it consists of taxation on payroll to the labour market institutions, T_t^{AM}, other personal income taxation, $T_t^{HouseHoldIncomeRest}$, weight charge on cars, $T_t^{CarWeight}$, corporate taxation, T_t^{Corp}, taxation on return on investments in pension funds, T_t^{PAL}, and the contribution to the public media, T_t^{Media}. Thus the direct taxation is given by:

$$\begin{aligned} T_t^{Direct} = \quad & T_t^{Income} + T_t^{AM} + T_t^{HouseHoldIncomeRest} \\ & + T_t^{CarWeight} + T_t^{Corp} + T_t^{PAL} + T_t^{Media} \end{aligned}$$

In this version $T_t^{CarWeight}$ & T_t^{PAL} follows GDP.

Note that a tax revenue labeled with no age subscript (T_t^{Income} i.e.) is the total tax revenue from all cohorts, while a revenue with an age subscript ($T_{a,t}^{Income}$ i.e.) is the average revenue from that cohort. The two variables are related by $T_t^{Income} = \sum_a n_{a,t} T_{a,t}^{Income}$.

The income tax $T_{a,t}^{Income}$ from a given cohort is given by:

$$\begin{aligned} T_{a,t}^{Income} = \quad & T_{a,t}^{Bot} + T_{a,t}^{Top} + T_{a,t}^{Muncipal} \\ & + T_{a,t}^{Property} + T_{a,t}^{Stock} + T_{a,t}^{IncomeRes} \end{aligned}$$

The income tax $T_{a,t}^{Income}$ consists of the revenue from basic and top income taxation $T_{a,t}^{Bot}$ & $T_{a,t}^{Top}$, revenue from income taxation in municipalities $T_{a,t}^{Muncipal}$ and taxation on property and stock income $T_{a,t}^{Property}$ & $T_{a,t}^{Stock}$. The rest of the income taxes are caught in the residual $T_{a,t}^{IncomeRes}$. The taxation on property and stock is for now exogenous.

The revenue from basic income taxation is given by:

$$\begin{aligned} T_{a,t}^{Bot} = \quad & t_t^{Bot} \cdot \left[Income_{a,t}^{Personal} + Income_{a,t}^{PosNetCapital} - PersAllowance_{a,t} \right] \\ & \cdot \left[1 + adjT_{a,t}^{Bot} \right] \end{aligned}$$

The basic income taxation is subject to the tax rate t_t^{Bot}, and is based on personal income, $Income_{a,t}^{Personal}$, positive capital gains, $Income_{a,t}^{PosNetCapital}$, and a personal allowance $PersAllowance_{a,t}$, which lowers the tax burden. The last part, $adjT_{a,t}^{Bot}$, is an adjustment factor to fit data.

[1] See http://www.skm.dk/skattetal/beregning/skatteberegning/skatteberegning-hovedtraekkene-i-personbeskatningen-2017

The revenue from top income taxation is given by:

$$T_{a,t}^{Top} = t_t^{Top} \cdot \left[Income_{a,t}^{Personal} + Income_{a,t}^{PosNetCapital}\right] \cdot IncomeShareAboveThreshold_{a,t}$$

The top income taxation is based on personal income and capital gains. It is subject to the tax rate t_t^{Top}. However, only income above a certain threshold, $IncomeShareAboveThreshold_{a,t}$, is taxed at this tax rate. $IncomeShareAboveThreshold_{a,t}$ is calibrated so as $T_{a,t}^{Top}$ fits data.

The municipal tax is given by:

$$T_{a,t}^{Muncipal} = t_t^{Muncipal} \cdot \left[Income_{a,t}^{Taxable} - PersAllowance_{a,t}\right] \cdot \left[1 + adjT_{a,t}^{Muncipal}\right]$$

The municipal taxation is based on taxable income with the personal allowance subtracted. It is subject to the tax rate $t_t^{Muncipal}$. $adjT_{a,t}^{Muncipal}$ is an adjustment factor. The income taxation besides the taxes mentioned above is caught in the residual, which is given by:

$$T_{a,t}^{IncomeRes} = w_{a,t} \cdot \frac{n_{a,t}^{Emp}}{n_{a,t}} \cdot adjT_t^{IncomeRes}$$

The residual income taxes depends on the wages $w_{a,t}$, the fraction of employed relative to the population $\frac{n_{a,t}^{Emp}}{n_{a,t}}$ and an adjustment factor $adjT_t^{IncomeRes}$.

The direct taxation not only consists of income taxation. It also consists of the taxation on payroll to the labour market institutions. This is given by:

$$T_{a,t}^{AM} = t_t^{AM} \cdot w_{a,t} \cdot \frac{n_{a,t}^{Emp}}{n_{a,t}} \cdot \left[1 + adjT_t^{AM}\right]$$

The taxation on payroll to the labour market institutions depends on wages, employed relative to the population, an adjustment factor $adjT_t^{AM}$ and a tax rate. All these factors are multiplied together.
Direct taxation also depends on other personal income taxation, which is given by:

$$T_{a,t}^{HouseHoldIncomeRest} = t_t^{CapPension} \cdot Pension_{a,t}^{RecieveCap} + Income_{a,t}^{Personal} \cdot adjT_t^{HouseHoldIncomeRest}$$

The main part of other personal income taxation is a tax on the money received from capital pensions at the payout time $Pension_{a,t}^{RecieveCap}$. This is subject to the tax rate $t_t^{CapPension}$. The remaining part of other personal income taxation follows personal income. $adjT_t^{HouseHoldIncomeRest}$ is an adjustment factor.

Corporate taxation is given by:

$$T_{sp,t}^{Corp} = T_{sp,t}^{Main} + T_{EXT,t}^{CorpNorth}$$

Corporate tax revenue consists of tax revenue from the extraction sector (indexed EXT), $T_{EXT,t}^{CorpNorth}$, and from the remaining private sector, $T_{sp,t}^{Main}$. The corporate tax is based on earnings before taxes, $EBT_{sp,t}$, and is subject to the tax rate $t_{sp,t}^{Corp}$. The tax revenue from a sector not concerned with extraction is:

$$T_{sp,t}^{Main} = t_{sp,t}^{Corp} \cdot EBT_{sp,t} + GVA_{sp,t} \cdot adjT_t^{Corp}$$

where it is understood that the index *sp* covers private sectors not concerned with extraction. $GVA_{sp,t}$ denotes the gross value added in the sector, which together with the adjustment factor $adjT_t^{Corp}$ensures data fit. Tax revenue from sectors concerned with extraction is given by:

$$T_{EXT,t}^{CorpNorth} = t_{EXT,t}^{Corp} \cdot EBT_{EXT,t} + GVA_{EXT,t} \cdot adjT_t^{CorpNorth}$$

The taxation of the extraction sector is subject to the tax rate $t_{EXT,t}^{Corp}$, and based on earnings before taxes in the sector, $EBT_{EXT,t}$. Gross value added $GVA_{EXT,t}$ and the adjustment factor $adjT_t^{CorpNorth}$ ensures that revenues fit data.

Finally, contributions to the public media is given by:

$$T_t^{Media} = T_t^{MediaPrPers} \cdot \sum_a n_{a,t}$$

It is estimated as a fixed amount payed by each person, $T_t^{MediaPrPers}$, multiplied by the population $\sum_a n_{a,t}$.

2.3.2 Income terms and allowances

Personal income is given by:

$$\begin{aligned} Income_{a,t}^{Personal} = [& w_{a,t} \cdot \frac{n_{a,t}^{Emp}}{n_{a,t}} + Trans_{a,t}^{LabMarket} + Trans_{a,t}^{Pension} + Trans_{a,t}^{RestTax} \\ & - Pension_{a,t}^{PayMain} - Pension_{a,t}^{PayATP} - Pension_{a,t}^{PayCap} + Pension_{a,t}^{RecieveMain} \\ & + Pension_{a,t}^{RecieveATP} - T_{a,t}^{AM}] \cdot \left[1 + adjIncome_{a,t}^{Personal}\right] \end{aligned}$$

$Trans_{a,t}^{LabMarket}$, $Trans_{a,t}^{Pension}$, $Trans_{a,t}^{RestTax}$ are defined in 2.4.1. $Pension^X$are all exogenous.

Taxable Income is given by:

$$\begin{aligned} Income_{a,t}^{Tax} = [& Income_{a,t}^{Personal} + Income_{a,t}^{NetCapital} - Allowance_{a,t}^{EITC} - Allowance_{a,t}^{Unemp} \\ & - Allowance_{a,t}^{EarlyRet} - Allowance_{a,t}^{Other}] \cdot \left[1 + adjIncome_{a,t}^{Tax}\right] \end{aligned}$$

Taxable income is therefore smaller than personal income as it includes negative capital income and a rank of allowances defined later.

Net Capital Income (for an average person in a given cohort) is simply given by:

$$Income_{a,t}^{NetCapital} = Income_{a,t}^{PosCap} - Income_{a,t}^{NegCap}$$

Positive and negative capital income are for now exogenous. They follow macro data from statistikbanken.dk and age profiles from registerdata. They will be modeled in a later version.
As seen above all capital income is part of taxable income and thereby the tax base for municipal taxation. Only positive net capital income (above a certain threshold) is part

of the tax base for base and top taxation. The modeling of this positive net capital income is difficult. This problem will be further investigated in a later version. For now the modeling is very simple, and given by:

$$Income_{a,t}^{PosNetCapital} = Income_{a,t}^{PosCap} \cdot 0.5$$

The personal allowance is the same for every (adult) person and follows the indexation of transfers $fGov_t$. The effect of the personal allowance is however not the same for all cohorts as some (few) persons do not have an income[2]. The personal allowance is therefore corrected by multiplying with the parameter $PopShareUsingPersAllowance_{a,t}$ which is close to 1:

$$Allowance_{a,t}^{Pers} = Allowance_{a,t}^{PersMax} \cdot PopShareUsingPersAllowance_{a,t}$$

$$Allowance_{a,t}^{PersMax} = Allowance_{a,t-1}^{PersMax} \cdot fGov_t + adjAllowance_{a,t}^{PersMax}$$

The earned income tax credit (Beskæftigelsesfradrag) is an allowance for people in employment. It is a bit tricky to model in a model with representative agents as the EITC has the properties of a negative marginal tax for people with low income and a negative lump sum tax for people with high income. In this model the EITC is determined by an average rate for each cohort, $Allowance_{a,t}^{AvgEITC}$, determined from registerdata. Furthermore the EITC is scaled (via the variable $adjAllowance_{a,t}^{EITC}$) to fit the total value of EITC from Statistikbanken:

$$Allowance_{a,t}^{EITC} = Allowance_{a,t}^{AvgEITC} \cdot \left[W_{a,t} \cdot \frac{n_{a,t}^{Emp}}{n_{a,t}} \right] \cdot \left[1 + adjAllowance_{a,t}^{EITC} \right]$$

Allowance for contribution to unemployment insurance, $Allowance_{a,t}^{Unemp}$, and allowance for contribution to early retirement, $Allowance_{a,t}^{EarlyRet}$, follow the contributions. The allowances are bigger than the contributions as allowance includes contribution and administration cost. Therefore $Allowance_t^{ExoUnemp}$ and $Allowance_t^{ExoEarlyRet}$ are above 1.

$$Allowance_{a,t}^{Unemp} = Allowance_t^{ExoUnemp} \cdot Cont_{a,t}^{Unemp}$$

$$Allowance_{a,t}^{EarlyRet} = Allowance_t^{ExoEarlyRet} \cdot Cont_{a,t}^{EarlyRet}$$

Other allowances include allowances for transport, clothes etc. The allowances are primarily related to employment and therefore modeled to follow employment:

$$Allowance_{a,t}^{Other} = Allowance_t^{ExoOther} \cdot \frac{n_{a,t}^{LabForce}}{n_{a,t}} \cdot fGov_t$$

[2] The personal allowance can be used by a spouse if a person has no income (and is married). This effect is not captured in the model.

2.3.3 Indirect taxation

The indirect taxes, $T_t^{Indirect}$, are given by excise duties, $T_t^{ExciseDuty}$, duties from car sales (registreringsafgifter), T_t^{Reg}, VAT, T_t^{VAT}, and production taxes, $T_t^{Production}$. Production taxes will be further specified in a later version. VAT and Duties are generated in the section "Taxes". Indirect taxes also includes any difference between customs ,T_t^{Cus}, and indirect taxes to the EU, T_t^{EU}.

$$T_t^{Indirect} = T_t^{VAT} + T_t^{ExciseDuty} + T_t^{Reg} + T_t^{Production} + T_t^{Cus} - T_t^{EU}$$

To ensure consistency between customs and indirect taxes to the EU an adjustment term is added in the following way:

$$T_t^{EU} = T_t^{Cus}\left(1 + adjT_t^{EU}\right)$$

2.3.4 Other government revenues

Other government revenues is the sum of many different revenues. The specific modeling is not yet complete. Other government revenues are currently given by:

$$\begin{aligned} GovRev_t^{Rest} = \; & T_t^{Bequest} + GovDepr_t + Cont_t \\ & + GovReceive_t^{Foreign} + GovReceive_t^{HHFirms} \\ & + GovRent_t + GovProfit_t \\ & + T_t^{Church} + GovRev_t^{Fit} \end{aligned}$$

Tax on bequest (kapitalskatter/arveafgift),$T_t^{Bequest}$, follow GDP and will be developed further in a later version . Revenues from the use of government capital[3], $GovDepr_t$, (afskrivninger/ bruttorestindkomst) are given by:

$$GovDepr_t = \sum_K P_{Pub,t-1}^{uK} \cdot K_{Pub,t-1}$$

P_{Pub}^{uK} is the user cost of capital in the public sector, simply defined as the depreciation rate, while K_{Pub} is the amount of capital in the public sector. This includes both types of capital (machinery and buildings).

Contributions, $Cont_t$, (Bidrag til social ordninger) is the sum of:

$$\begin{aligned} Cont_t = \; & Cont_t^{Unemp} + Cont_t^{EarlyRet} + Cont_t^{FreeRest} \\ & + Cont_t^{Mandatory} + Cont_t^{CivilServants} \end{aligned}$$

All contributions follow demographic data and participation characteristics through a relation of the form:

$$Cont_{a,t}^X = r_t^X \cdot \frac{n_{a,t}^{LabForce}}{n_{a,t}} \cdot fGov_t,$$

where r_t^X denotes the rate of contributions. X equals the set of contributions (contributions to early retirement, $Cont_t^{EarlyRet}$ (bidrag til efterløn), free contributions, $Cont_t^{FreeRest}$ (øvrige frivillig bidrag), mandatory contributions, $Cont_t^{Mandatory}$ (obligatoriske bidrag), and contributions to civil servants pensions, $Cont_t^{CivilServants}$ (bidrag til Tjenestemandspension)).

[3] Use of government capital is part of the price on public consumption. Therefore it is both a cost and a revenue when the government use its own capital to produce a public good.

Payments from foreign countries, $GovReceive_t^{Foreign}$, payments from households and domestic firms, $GovReceive_t^{HHFirms}$, and profits from public corporations ,$GovProfit_t$, are all calibrated to match their respective shares of GDP. For example, given GDP and government profit's share of GDP, $GDPShare_t^{GovProfit}$, $GovProfit_t$ is calculated as:

$$GovProfit_t = GDPShare_t^{GovProfit} \cdot GDP_t$$

The landrent, $GovRent_t$, depends on the gross value added in the extraction sector, and is given by:

$$GovRent_t = t_t^{Rent} \cdot GVA_{EXT,t}$$

Tax revenue from the church tax follows the same tax base as municipal taxation and is (at a personal level) given by:

$$T_{a,t}^{Church} = t_t^{Church} \cdot \left[Income_{a,t}^{Taxable} - PersAllowance_{a,t}\right] \cdot \left[1 + adjT_t^{Church}\right]$$

Lastly, $GovRev_t^{Fit}$ is a variable that secures that $GovRev_t$ fits real data. It shall be close to zero.

2.4 Public expenditures

Public expenditures is the sum of public consumption, government income transfers, government investments, subsidies and other public expenditures.

$$GovExp_t = G_t + Trans_t + I_t^{Gov} + GovSub_t + GovExp_t^{Rest}$$

Public consumption, G_t, follows GDP_t. G_t can be further developed to take demographic changes, healthy ageing and so on into account. Only the total expenditures used on public consumption is determined in this section. The price index together with the deliveries to government consumption is determined in the Input/Output section. The modeling of the remaining expenditures are described in the next subsections.

2.4.1 Government income transfers

Government income transfers, $Trans_{a,t}$, are defined over age and represents the average transfers a person receives for a given age and time. $Trans_t$ (in the code it is called Trans(aTot,t) or Trans('tot',t)) represent the total expenditure to transfers such that $Trans_t = \sum_a Trans_{a,t} \cdot n_{a,t}$. Government income transfers are given by:

$$Trans_{a,t} = Trans_{a,t}^{LabMarket} + Trans_{a,t}^{Pensions} + Trans_{a,t}^{Other}$$

Transfer payments related to labor market status, $Trans_{a,t}^{LabMarket}$, follows:

$$Trans_{a,t}^{LabMarket} = Trans_{a,t}^{Unemp} \cdot \frac{n_{a,t}^{Unemp}}{n_{a,t}} + Trans_{a,t}^{LabMarketRest} \cdot \frac{n_{a,t}^{Olf}}{n_{a,t}}$$

$n_{a,t}^{Unemp}$and $n_{a,t}^{Olf}$ follows the definitions in the labor market. The labor market groups correspond to definitions in the Ministry of Finance so as $Trans_{a,t}^{Unemp}$ correspond to the number of gross unemployed.
Transfer payments to pensions are given by:

$$\begin{aligned} Trans^{Pensions}_{a,t} = \quad & Trans^{OldAgePens}_{a,t} \cdot \frac{n^{OldAgePens}_{a,t}}{n_{a,t}} + Trans^{FortidsPens}_{a,t} \cdot \frac{n^{FortidsPens}_{a,t}}{n_{a,t}} \\ & + Trans^{EarlyRet}_{a,t} \cdot \frac{n^{EarlyRet}_{a,t}}{n_{a,t}} \\ & + Trans^{CivilServants}_{a,t} \cdot \frac{n^{CivilServants}_{a,t}}{n_{a,t}} \\ & + Trans^{PensOther}_{a,t} \cdot \frac{n^{OldAgePens}_{a,t} + n^{EarlyRet}_{a,t}}{n_{a,t}} \end{aligned}$$

Old age pension (Folkepension), Førtidspension, Early retirement (Efterløn), Civil Servants pensions (Tjenestemandspension) and other pensions. As it is seen from the equation the total expenditures to the transfers depends on the number of individuals enrolled to a given income transfer. Furthermore, it depends on the rate for a given income transfer. For now, the rate is the same for all age groups. With solid micro data this can be changed for relevant groups of income transfers. The rates follows the indexation of transfers, $fGov_t$. Thereby a given rate for an income transfer is given by:

$$Trans^X_{a,t} = fGov_t \cdot Trans^X_{a,t-1} + Adj^X_{a,t}$$

where the adjust terms, $Adj^X_{a,t}$, is included to calibrate to actual data. It is assumed that "satspuljen" is used to increase transfers proportionally. The indexation of transfers follows wages of the employed with a two year lag:

$$fGov_t = \frac{W_{t-2}L_{t-2}/Employed_{t-2}}{W_{t-3}L_{t-3}/Employed_{t-3}}$$

Other Transfer payments are given by:

$$\begin{aligned} Trans^{Other}_{a,t} = \quad & Trans^{Family}_{a,t} + Trans^{Green}_{a,t} \\ & + Trans^{HouseOther}_{a,t} \cdot \frac{n_{a,t} - n^{OldAgePens}_{a,t} - n^{WornPens}_{a,t}}{n_{a,t}} \\ & + Trans^{HousePens}_{a,t} \cdot \frac{n^{OldAgePens}_{a,t} + n^{WornPens}_{a,t}}{n_{a,t}} \\ & + Trans^{RestTaxable}_{a,t} + Trans^{RestNonTaxable}_{a,t} + \frac{Trans^{Fit}_t}{\sum_{a=1}^{A} n_{a,t}} \end{aligned}$$

Transfer payments to families with children, $Trans^{Family}_{a,t}$, (Børnefamilieydelse) follows the number children. It is assumed that all adults in the age between 18 and 55 have children and receive the same amount in "Børnefamilieydelse". Transfer payments to compensate for green taxes, $Trans^{Green}_{a,t}$, (Grøn check) follows the adult population. The rate is constant in nominal terms. Housing support to population not enrolled in old age pension or worn pension, $Trans^{HouseOther}_{a,t}$, (Boligstøtte) and Housing support to population inrolled in old age pension or worn pension, $Trans^{HousePens}_{a,t}$ (Boligydelse) follows the two population groups. Other taxable transfer payments, $Trans^{RestTaxable}_{a,t}$, (Øvrige skattepligtige overførsler) and Other non-taxable transfer payments, $Trans^{RestNonTaxable}_{a,t}$, (Øvrige ikke-skattepligtige overførsler) follow the population and all the rates follow the underlying inflation and growth. $Trans^{Fit}_t$ is a variable that secures that $Trans_t$ fits real data. It shall be close to zero.

2.4.2 Government investments

Government investments are described in the chapter concerning public production.

2.4.3 Government subsidies and other expenditures

Government subsidies are given by subsidies for products and production minus subsidies received from the EU:

$$GovSub_t = Sub_t^{Product} + Sub_t^{Production} - Sub_t^{EU}$$

Given GDP_t, $Sub_t^{Production}$ and Sub_t^{EU} are calibrated to match their respective shares of GDP from real data. $Sub_t^{Product}$ is expanded upon in the section “Taxes”. The remaining government expenditures are given by:

$$\begin{aligned} GovExp_t^{Rest} = \quad & Gov_t^{LandRights} + Gov_t^{PaymentForeign} \\ & + Gov_t^{PaymentHH} + Gov_t^{PaymentFirms} \end{aligned}$$

$Gov_t^{LandRights}$ covers expenditures to purchase land and rights, $Gov_t^{PaymentForeign}$ is payments to foreign countries, $Gov_t^{PaymentHH}$ is payments to households, and $Gov_t^{PaymentFirms}$ is payments to domestic firms. These variables are all calibrated to match their observed share of GDP.

2.5 Structural budget balance

The structural budget balance is a key parameter in the Danish economy as it is used by the ministry of finance to secure that the public budget is fiscal sustainable. One of the key elements in the budget law (Budgetloven) is that the deficit on the structural budget balance cannot exceed ½ pct. GDP. Thereby, it is crucial how the structural budget balance is calculated as it sets the framework for fiscal negotiations regarding the public budget. The ministry of finance has developed a very detailed way to calculate the structural budget balance. The structural budget balance in MAKRO is calculated with the same methodology. However, not all details from the method used by the ministry of finance are implemented in the structural budget balance in MAKRO.

The structural budget balance has the same structure as the real budget balance. Thereby, every component in the real budget balance has a corresponding structural level. The structural level is calculated by correcting the real value. To do so two approaches has been used. The main part of the variables is corrected with respect to the business cycle gap. The remaining is corrected by using a 7-year average value of the real value.

The business cycle gap is calculated by weighing the Gross Value Added gap (by 40 pct.) and the unemployment gap (by 60 pct.). The business cycle gap is then used to correct the real values for public revenues and expenditures. However, different public revenues and expenditures are not affected by business cycles in the same way. Therefore, the ministry of finance has estimated a large number of elasticities explaining how a given revenue/expenditure is affected by a business cycle gap. These elasticities are used to calculate the structural value for revenues and expenditures in the following way:

$$ValueX_t^{Structural} = ValueX_t^{Real} * \left(1 - elasticity^{ValueX} * BusinessCycleGap_t\right)$$

The elasticities used in calculation of the structural values are listed in the tables in the appendix for government variables .

Variation in some remaining revenues and expenditures cannot be explained by the business cycle gap. Therefore the structural value for those variables is calculated by other means. The ministry of finance has developed specific methods for some individual revenues; for instance the revenue from taxation on return on pensions. Those specific methods are not (yet) implemented in MAKRO. However, a group of structural revenues and expenditures are calculated by taking a 7-year average over the real values. This method is implemented in MAKRO. The tables in the appendix for government variables indicate which structural values that calculated by using a 7-year average.

2.6 Fiscal sustainability indicator

The fiscal sustainability indicator, in the model called HBI, is calculated and defined by the standard definition. I.e. the fiscal sustainability indicator is equal to the net present value of all government revenues minus expenditures minus the initial government net debt relative to the net present value of GDP for infinite time periods:

$$HBI = \frac{\sum\limits_{t=2018}^{\infty} \left[\prod\limits_{tt=2018}^{t} \left[\frac{1}{1+iGovDebt_{tt}}\right] * vGovPrimBalance^{*}_{t} \right] + vGovWealth_{2018}}{\sum\limits_{t=2018}^{\infty} \left[\prod\limits_{tt=2018}^{t} \left[\frac{1}{1+iGovDebt_{tt}}\right] * vGDP_{t} \right]}$$

Net present values are calculated by using the rate on government debt. Revaluation on wealth and net interests that occur when the rate on government debt and assets are not equal are considered as revenues/expenditures in the indicator calculation, i.e. $vGovPrimBalance^{*}_{t} = vGovPrimBalance_{t} + vGovReval_{t} + vGovNetInterest_{t} - iGovDebt_{t} * vGovWealth_{t-1}$. It is assumed that the primary budget balance and GDP is constant (corrected from underlying growth and inflation) from year 2200 and onwards.

3 The firms

In addition to the public sector (pub) there are eight private sectors, subscript sp, in the economy: agriculture (agr), construction (con), energy provision(ene), extraction (ext), housing (hou), manufacturing (man), sea transport (sea) and services (ser). The firms in a given sector maximize the value of their stocks. This is equivalent to each sector maximizing the current discounted value of dividends. Solving this problem requires both cost minimization and setting an optimal price. As argued in the pricing chapter these two problems could be separated into two sub-sectors - an intermediate goods sector producing and a whole-sale sub-sector buying, selling, and setting prices. In the documentation (and code) the production and price setting decisions are separated where the cost minimization is given in this chapter and setting the optimal price is described in the pricing chapter.

Production in every sector uses labor, capital, and materials. In determining these factor inputs relative prices play a role. The demand for each private sector's production, $Y_{sp,t}$, as well as price index for material inputs, $P^{R}_{sp,t}$, are given in the Input-/Output chapter. The cost per effective unit of labor, $P^{L}_{sp,t}$, is given in the labor market chapter, and the user cost for capital[4], $P^{uK}_{k,sp,t}$, is given in this chapter. User cost depends on the investment prices, $P^{I}_{i,sp,t}$, which is also determined in the Input-/Output chapter.[5]

Production technology is a nested CES with the structure shown below. This section delivers two of the five major demand components - namely material inputs,$R_{sp,t}$, and investments, $I_{i,s,t}$ - to the Input/-Output chapter as well as aggregated labor demand, L_t, to the labor market chapter. The forward looking expectations in the user cost equation makes the factor demand decisions dynamic.

NOTE: In the model equations capital utilization exists. These are an integrated part of the production, but they are not described in this section. This implies that the equations in this section will not exactly match the equations in the code. The basic mechanisms are unchanged by introducing capital utilization and this chapter can still be read to give an understanding of the basics of the current formulation.

3.1 The value of the firm

It is assumed that all firms are owned by stockholders. Stocks from privately owned firms can be seen as non-issued stocks held by the owners. The total value of the equities of all firms, $Equity_t$, is given by the sum of discounted free cash flows to equity[6], $FCFE_t = \sum_s FCFE_{sp,t}$, which can be written as a difference equation:

$$Equity_t = \sum_{n=1}^{\infty} \frac{FCFE_{t+n}}{\prod_{m=1}^{n}\left(1 + r^{firms}_{t+m}\right)} = \frac{FCFE_{t+1}}{1 + r^{firms}_{t+1}} + \frac{Equity_{t+1}}{1 + r^{firms}_{t+1}}$$

The interest rate of the firm is the risk free interest rate plus a premium[7] $r^{firm}_t = r^{RF}_t + risk^{prem}_t$. The dividends in a specific sector is given by:

[4]There are two types of capital denoted with subscript k. The set k consists of iM and iB. iM being machinery and iB being buildings. k is part of the entire investment set, which consists of iM, iB and Inventory. With Invt being inventory investments.

[5]Note that material inputs can be referred to as $R_{rp,t}$ and its price $P^{R}_{rp,t}$ where $rp \equiv sp$. The reason is that in the Input/-Output there are deliveries from sectors to sectors. Here the receiving sector is called r and the delivering called s.

[6]Free cash flow to equity is a metric of how much cash can be distributed to the equity shareholders as dividends or stock buybacks.

[7]In this version of the model the average bond rate (iwbz from ADAM) plus a risk premium (exogenously set to 0.02).

$$FCFE_{sp,t} = \left(1 - t^{Corp}_{sp,t}\right) EBT_{sp,t} + \sum_k \delta^{Book}_{k,sp,t} K^{book}_{k,sp,t-1} - P^I_{sp,t} I_{sp,t} + \left(FirmDebt_{sp,t} - FirmDebt_{sp,t-1}\right)$$

where $t^{Corp}_{sp,t}$ is the corporate tax rate, $EBT_{sp,t}$ is the bookkeeper value of earnings before taxes, $\delta^{book}_{k,sp,t}$ is the tax deductible depreciation rate, $K^{book}_{k,sp,t-1}$ is the book value of capital, $P^I_{sp,t} I_{sp,t} = \sum_i P^I_{i,sp,t} I_{i,sp,t}$ is the value over all types of investments, and $FirmDebt_{sp,t}$ is the value of the firms debt. The firm's earnings after tax is its earnings before taxes times a tax rate and corrected for being able to deduct depreciation. This earnings can be used for investments, to pay of debt or as free cash flow to equity used to pay out dividends or buy back stocks. In this equation the free cash flow to equity is the residual as the firm's debt follows the value of the capital stock and investments are made to maximize the discounted value of the free cash flow to equity. It is possible to have negative free cash flow to equity in a given year - this is interpreted as issuing stocks.

Firm debt is an exogenous share, $\mu^{FirmDebt}_{sp,t}$, of the value of the capital stock[8]:

$$FirmDebt_{sp,t} = \mu^{FirmDebt}_{sp,t} \sum_k P^I_{k,sp,t} K_{k,sp,t}$$

The firm benefits from a favorable tax treatment of capital depreciation with a tax deductible depreciation rate, which can be higher than the actual rate of depreciation. Therefore there is a nominal aggregate which accumulates and is the source of the tax benefit. We call it the “book value” of capital and it is given by:

$$K^{book}_{k,sp,t} = \left(1 - \delta^{book}_{k,sp,t-1}\right) K^{book}_{k,sp,t-1} + P^I_{k,sp,t} I_{k,sp,t}$$

The bookkeeper earnings before taxes is given by:

$$EBT_{sp,t} = \frac{P^Y_{sp,t} Y_{sp,t}}{1 + t^Y_{sp,t}} - P^L_{sp,t} L_{sp,t} - P^R_{sp,t} R_{sp,t} - \sum_k \delta^{Book}_{k,sp,t} K^{book}_{sp,t-1} - r^{firms}_t FirmDebt_{sp,t-1}$$

where $t^Y_{sp,t}$ is the production tax rate. In this model version it is assumed that firms do not own bonds or stocks in other firms these are owned directly by domestic and foreign households.

3.2 Investments and installation costs

Investments in capital used in production, $I_{k,sp,t}$, is given by the dynamic identities for capital accumulation:

$$I_{k,sp,t} = K_{k,sp,t} - (1 - \delta_{k,sp,t}) K_{k,sp,t-1}$$

where $\delta_{k,sp,t}$ is the depreciation rate.

Inventory investments, $I_{invt,sp,t}$, are in this version of the model proportional to production:

$$I_{invt,sp,t} = \mu^{Invt}_{sp,t} Y_{sp,t}$$

[8] To make the model tractable we assume that the most disaggregated levels in the model is the micro levels. This implies that the value of capital is measured by its investment price. In the national accounts our capital types are aggregates and the capital stock will have a different composition than the investments. This means that due to composition effects the price of the capital stock will differ from the investments for a given aggregate and our capital value will not match the one from the national accounts.

Inventory investments are a data driven detail. We do not have a model of inventories. Therefore inventory investment does not accumulate, nor does it contribute to production. It merely acts as a drain on resources in order to match the model with national accounts data, where it is a small fraction of total spending (less than one half of one percent).

A part of user cost is quadratic installation costs for each used capital type $KInstCost_{k,sp,t}$. These are, however, not an explicit expenditure for the firm - they are measured in lost production. Gross production, $Y^{gross}_{sp,t}$, i.e. production including these installation costs are given by:

$$Y^{gross}_{sp,t} = Y_{sp,t} + \sum_k KInstCost_{k,sp,t} = Y_{sp,t} + KInstCost_{iM,sp,t} + KInstCost_{iB,sp,t}$$

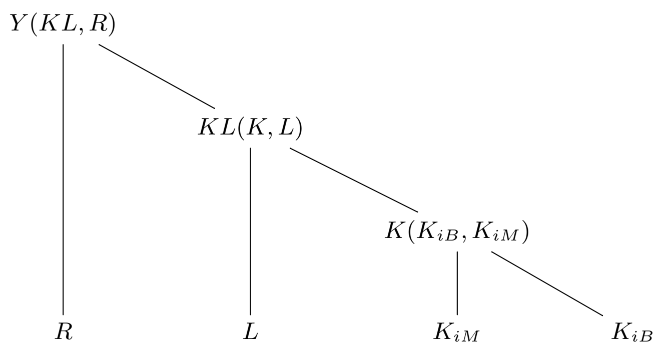
The installation costs are given by[9]:

$$KInstCost_{k,sp,t} = \mu^{KInstCost}_{k,sp,t} \frac{(I_{k,sp,t} - \xi K_{k,sp,t-1})^2}{K_{k,sp,t-1}}$$

where $\mu^{KInstCost}_{sp,t}$ is an exogenous calibrated parameter, $\xi = \delta_{k,sp,t} + g_t$ and g_t is the Harrod neutral steady state growth rate.

3.3 Production inputs: Materials, labor and capital

Every sector, s, has a nested CES production function with input of materials, $R_{sp,t}$, capital (excluding inventories), $K_{k,sp,t-1}$, and labor, $L_{sp,t}$. Capital excluding inventories, $K_{k,sp,t-1}$, consists (in this model version) of building capital, $K_{iM,sp,t-1}$, machinery capital, $K_{iB,sp,t-1}$. The nesting structure is shown below:



It is assumed that the firms in a given sector tries to maximize the discounted value of their dividends subject to the capital accumulation of real and book value capital and subject to the sector specific production function. The problem of the firm is detailed in the appendix.

In the top nest the firms choose between inputs of materials, $R_{sp,t}$, and an aggregate of capital and labor, $KL_{sp,t}$:

$$R_{sp,t} = \mu^{YR}_{sp,t} Y^{gross}_t \left(\frac{P^R_{sp,t}}{P^{Y0}_{sp,t}} \right)^{-\sigma^Y_p}$$

$$KL_{sp,t} = \mu^{YKL}_{sp,t} Y^{gross}_t \left(\frac{P^{KL}_{sp,t}}{P^{Y0}_{sp,t}} \right)^{-\sigma^Y_{sp}}$$

[9] The quadratic investment is 0 in steady state where $K_{k,sp,t} = (1 + g_t) K_{k,sp,t-1}$ and $I_{k,sp,t} = \xi K$. This implies that there are no quadratic installation costs affecting the user cost in steady state. The further away from steady state investments the larger the user cost.

where $\mu^{YR}_{sp,t}$ and $\mu^{YKL}_{sp,t}$ are calibrated share parameters, σ^{Y}_{sp} is the elasticity of substitution between materials and other production inputs, $P^{R}_{sp,t}$ and $P^{KL}_{sp,t}$ are sector specific CES price indices for the materials and the KL aggregates, and $P^{Y0}_{sp,t}$ is the marginal cost of producing one more unit of output. The elasticity of substitution, σ^{Y}_{sp}, is in this model version set to 0.67 for all sectors. This is roughly in line with the estimates from Thomas Thomsen [REFERENCE].

In the next nest firms choose between labor, $L_{sp,t}$, and capital, $K_{sp,t}$:

$$L_{sp,t} = \mu^{LKL}_{sp,t} \frac{KL_{sp,t}}{e^{L}_{sp,t}} \left(\frac{P^{L}_{sp,t}/e^{L}_{sp,t}}{P^{KL}_{sp,t}} \right)^{-\sigma^{KL}_{sp}}$$

$$K_{sp,t-1} = \mu^{KKL}_{sp,t-1} KL_{sp,t} \left(\frac{P^{uK}_{sp,t-1}}{P^{KL}_{sp,t}} \right)^{-\sigma^{KL}_{sp}}$$

where $\mu^{LKL}_{sp,t}$ and $\mu^{KKL}_{sp,t}$ are calibrated share parameters, σ^{KL}_{sp} is the elasticity of substitution between capital and labor, $P^{L}_{sp,t}$ is the wage cost per effective unit of labor, $P^{uK}_{sp,t}$ is the CES price index for the user cost of capital, and e^{L}_{t} is the macro labor productivity (in the code called prod_s) following the Harrod-neutral growth rate. In the current version of the model lagged capital is used in the production function. This means that the capital is chosen a period before and substitutes to labor in the next period. The elasticity of substitution is in this model version set to 0.5 for all sectors. This is somewhat higher than in ADAM where the elasticity between machinery capital and labor varies among sectors on a slightly lower level, and the elasticity to building capital is set to zero. Note that the small open economy nature of the model determines that the supply of capital is exogenous and horizontal at the world interest rate.

In the final production nest firms choose between the different types of capital i.e. buildings, $K_{iB,sp,t}$, and machinery, $K_{iM,sp,t}$:

$$K_{k,sp,t} = \mu^{KK}_{k,sp,t} K_{sp,t} \left(\frac{P^{uK}_{k,sp,t}}{P^{uK}_{sp,t}} \right)^{-\sigma^{KK}_{sp}}$$

where $\mu^{KL}_{sp,t}$ are calibrated share parameters, σ^{K}_{s} is the elasticity of substitution between the different types of capital, and $P^{uK}_{k,sp,t}$ is the user cost for the different types of capital. The elasticity of substitution is in this model version set to 0.25 for all sectors. This is quite arbitrary and must be further investigated.

The CES prices are simply given as:

$$P^{KL}_{sp,t} = \frac{P^{uK}_{sp,t-1} K_{sp,t-1} + P^{L}_{sp,t} L_{sp,t}}{KL_{sp,t}}$$

$$P^{uK}_{sp,t} = \frac{\sum_k P^{uK}_{k,sp,t} K_{k,sp,t}}{K_{sp,t}} = \frac{P^{uK}_{iM,sp,t} K_{iM,sp,t} + P^{uK}_{iB,sp,t} K_{iB,sp,t}}{K_{sp,t}}$$

The marginal cost of producing one more unit of output is also a CES price index:

$$P^{Y0}_{sp,t} = \frac{P^{R}_{sp,t} R_{sp,t} + P^{KL}_{sp,t} KL_{sp,t}}{Y^{gross}_{sp,t}}$$

The price index for materials, $P^{R}_{sp,t} = P^{R}_{sp,t}$, are given in the Input-/Output chapter, the wage cost per effective unit of labor is $P^{L}_{sp,t}$, given in the labor market chapter. User cost is given in the next section.

3.4 User cost

User cost is derived in solving the problem of the firm as detailed in the appendix and is given by:

$$\begin{aligned} P^{uK}_{k,sp,t} \quad = \quad & \frac{1+r^{firms}_{t+1}}{1-t^{Corp}_{sp,t+1}} P^{sK}_{k,sp,t} - \frac{1-\delta_{k,sp,t+1}}{1-t^{Corp}_{sp,t+1}} P^{sK}_{k,sp,t+1} \\ & + \frac{t^{Corp}_{sp,t+1}}{1-t^{Corp}_{sp,t+1}} r^{firms}_{t+1} \mu^{FirmDebt}_{sp,t+1} P^{I}_{k,sp,t+1} \\ & - P^{Y}_{sp,t+1} \mu^{KInstCost}_{k,sp,t+1} \left(\left(\frac{I_{k,sp,t+1}}{K_{k,sp,t}} \right)^2 - \xi^2 \right) \end{aligned}$$

where r^{firms}_t is the interest rate of the firms, $t^{Corp}_{sp,t}$ is the corporate tax rate, $P^{sK}_{k,sp,t-1}$ is the shadow price of capital, $\delta_{k,sp,t}$ is the depreciation rate, $\mu^{FirmDebt}_{sp,t}$ is the debt share of the firms, $P^{I}_{k,sp,t}$ is the investment price and g_t is the Harrod neutral steady state growth rate. Every term on the right hand side is divided by $1 - t^{Corp}_{sp,t}$ since the surplus of the firm is subject to this tax rate and the marginal gain by increasing production is lowered proportionally. The first term is the cost of acquiring capital. This is simply the shadow price of capital forward discounted since it must be acquired one period before it is used in production. The second term is the resale value of the capital i.e. the shadow price of capital next period after depreciation. The third term is the tax shield. Interest rate expenses are tax deductible. The fourth term is how much a larger capital stock lowers the installation cost next period. It is a second order effect. The first order effect from investment costs are in the shadow price.

The shadow price is given by:

$$\begin{aligned} P^{sK}_{k,sp,t} \quad & = P^{I}_{k,sp,t} + \left(1 - t^{Corp}_{sp,t}\right) P^{Y}_{sp,t} \mu^{KInstCost}_{k,sp,t} 2 \left(\left(\frac{I_{dix,sp,t}}{K_{dix,sp,t-1}} \right) - \xi \right) \\ & - P^{I}_{k,sp,t} P^{sKbook}_{k,sp,t} \end{aligned}$$

where the first term is simply the investment price, the second term is the marginal installation cost (this cost is part of lost earnings and expenses are hence after tax) and the third term is the discounted value of all future tax deductible depreciation. The term $P^{sKbook}_{k,sp,t}$ is defined as:

$$P^{sKbook}_{k,sp,t} = \frac{1}{1 + r^{firms}_{t+1}} \left[\left(1 - \delta^{book}_{k,sp,t+1}\right) P^{sKbook}_{k,sp,t+1} + t^{Corp}_{sp,t+1} \delta^{book}_{k,sp,t+1} \right]$$

where $\delta^{book}_{k,sp,t}$ is the tax deductible depreciation rate. Behind this formulation lies the assumption that investments are not tax deductible in the same period they are made as they do not depreciate until next period. This should be changed in a future model version.

3.5 Labor in efficiency units and heads

If workers in all sectors supplied the same amount of labor in efficiency units, $L_{s,t}/n^{L}_{s,t}$, then employment in heads in a sector, $n^{L}_{s,t}$, would simply be proportional to units of efficient labor in the sector, $L_{s,t}$. The average wage per worker, $(w_t L_{s,t})/n^{L}_{s,t}$, in the different sectors can, however, vary even though the wage per effective unit of labor, w_t, is the same. Some sectors simply employ people who supply more effective units of labor than others. Our model does not explain why sectors differ. We simply use correction terms, $\lambda^{nL}_{s,t}$, to match the data:

$$n^{L}_{s,t} = \frac{\lambda^{nL}_{s,t} L_{s,t}}{\sum_s \lambda^{nL}_{s,t} L_{s,t}} \left(n^{Employed}_t + n^{CrossBorder}_t \right)$$

This formulation ensures that total sectoral employment is given by the employed household members and cross border workers (incl. persons from the Danish shadow economy) i.e. by $\sum_s n^L_{s,t} = n^{Employed}_t + n^{CrossBorder}_t$. The linkage between total employment in heads and in efficiency units is given by $L_t = e^L_t \left(n^{Employed}_t + n^{CrossBorder}_t\right)$, where e^L_t is the average productivity of workers given by $e^L_t = \left(\sum_a e^L_{a,t} n^{Employed}_{a,t}\right) / n^{Employed}_t$. It depends on the age distribution, but it does not depend on the sectoral distribution. It is assumed that cross border workers have the same average productivity as workers from Danish households.

If one sector increases its labor demand, $L_{s,t}$, relative to the other sectors it will increase its employment, $n^L_{s,t}$. As age-dependent productivity, $e^L_{a,t}$, is exogenous, total labor supply in efficiency units, L_t, is only affected by sectoral changes if total employment, $n^{Employed}_t + n^{CrossBorder}_t$, is affected. This means that moving demand (and hence people) from one sector to another does not change total supplied efficiency units per worker. Technically moving people to more sectors with a higher supply of efficiency units per worker will increase the denominator in the equation above and all sectors will experience a decrease in supplied efficiency per worker to maintain an unchanged average.

3.6 Data and calibration

The sector specific capital stocks of equipment and structures are matched to fit national accounts data from ADAMs databank. Material and labor input in production are matched to input from the national accounts[10] by calibrating relevant share parameters. The parameters for the real depreciation rate and the inventory investment ratio to output are statically calibrated to match investments given capital stock and output.

As usercost depends on future investments and lagged capital stock is used in the production all parameters in the production function are dependent on future variables. The parameters in the equations for materials, KL-aggregate, labor, K-aggregate and the types of capital are set to match data for material input, the labor costs and the disaggregated capital stocks taken from the national accounts. Calibrated parameters are projected constant. The price index for the KL-aggregate and the usercost is set to 1 in the calibration year and it is assumed that these prices historically have been Paasche chain indices. This assumption is necessary to separate price and productivity increases and identify the parameters.

In this version of the model the value of the firm is calculated on the basis of current and future cash flows to equity and a discount rate for the firms and not matched to an estimate of actual firm value. The current and future cash flows to equity is calculated according to the model equations and not matched to an estimate from data. The free cash flow depends on the effective corporate tax, the real and book value rates of depreciation, share of firm debt, usercost, other costs and revenue. The usercost and book value of depreciation have no non-theoretical data source to be matched to, but the other costs and revenue are matched to the relevant posts in the production data. The costs directly as mentioned above and the revenue by matching the value of all demand components and sectoral prices.

It has been hard to calibrate a model with very volatile (and some times sector specific negative) investments and quadratic investment costs. In order to temporarily sidestep this difficulty the quadratic investment costs are turned of in the calibration. This should be revised in a later version of the model.

[10]It is not important if values or quantities are matched here as either prices or values are matched later. Having matched two of values, quantities and prices - the third is given by the identity that values are prices times quantities.

4 Public Production

Public production is central to the planning of the government budget. It determines the price of public consumption taking public investments into account and also determines public employment. The following points characterizes the modeling of public production:

- For the modeling of public production, public consumption can be seen as exogenous. In the long run nominal public consumption follows demographic development and in the short run real public consumption excluding depreciation is exogenous.
- In the long run the nominal capital to output ratio for the different types of capital are constant or follows an exogenous trend. The nominal shares of material and labor inputs are also constant in the long run. This implies that in the long run the public production behaves as a Cobb-Douglas function with a possible exogenous trend for capital.
- In the planning horizon public investments are exogenous while nominal public material and labor inputs relative to production excluding depreciation are held constant as in the long run.
- The public wage follows the private wage, but can be exogenously corrected. As hours worked in the public sector follows the private sector it applies for both heads and hours.
- Public production is measured by the input method and there is no productivity growth in the public sector.

In the subsections below it is described how overall public production, production inputs and its price index are determined.

4.1 Determining public production

The public production, $Y_{pub,t}$, is given in the Input/Output system as the sum of demand components drawing on public production. There are only three IO-cells drawing on public production: 1) public production to public consumption, $G\_\_y_{pub,pub,t}$, 2) public sales to private consumption of service, $C\_\_y_{cSer,pub,t}$, and 3) public direct investments to machinery, $I\_\_y_{iM,pub,t}$. In the planning horizon public sales, $vPublicSales_t$, and public direct investments, $vPublicDirectInvestment_t$, are exognized and together with the public price index they determine $C\_\_y_{cSer,pub,t}$ and $I\_\_y_{iM,pub,t}$. Real public consumption excluding depreciations, $GxDepr_t$, are used as exogenous input in the planning period. The variable is defined in values and in quantities as a Laspeyres chain index:

$$vGxDepr_t = vG_t - vGovDepr_t$$

$$vGovDepr_t = \sum_k P_{k,t}^{KDepr\_pub} K_{k,pub,t-1}$$

$$P_{t-1}^{GxDepr} GxDepr_t = P_{t-1}^{G} G_t - \sum_k P_{k,t-1}^{KDepr\_pub} K_{k,pub,t-1}$$

where vG_t is the value of public consumption, $vGovDepr_t$ is the value of public consumption, $P_{k,t}^{KDepr\_pub}$ is the cost of depreciation per unit of capital of type k, $K_{k,pub,t}$, is the ultimo capital stock of type k in the public sector, P_t^{GxDepr} and P_t^{G} are price indices.

In the planning period real public consumption, G_t , is given by previous periods prices and capital stock as well as the exogenized $GxDepr_t$. Public production to public consumption, $G\_y_{pub,pub,t}$, is a fixed share of real public consumption, G_t. In the planning period the exogenous variables $GxDepr_t$, $vPublicSales_t$ and $vPublicDirectInvestment_t$ together with the endogenous price index for public production, $P^{Y}_{pub,t}$, determines both real and nominal public production. The price of depreciations per unit of capital, $P^{KDepr\_pub}_{k,t}$, is determined in this chapter, while the price of public consumption, P^{G}_{t} , is determined in the Input/Output section as it also depends on the price of the private inputs.

After the planning horizon $GxDepr_t$ is endogenous and G_t is exogenized and set to follow the demographic development. The values of public sales and public direct investments are after the planning period endogenously given by a fixed factor, $\mu^{PublicDirectInvestment2GVA}_{t}$, times nominal total gross value added, $vGrossValAdd_t$:

$$vPublicDirectInvestment_t = \mu^{PublicDirectInvestment2GVA}_{t} vGrossValAdd_t$$

4.2 The public Cobb-Douglas production function

The public production has three inputs: materials, labor and capital. In the planning horizon public investments, $I_{i,pub,t}$, are exogenously given and the capital stocks, $K_{pub,k,t}$, are given by the capital accumulation identities:

$$I_{k,pub,t} = K_{k,pub,t} - (1 - \delta_{k,pub,t})\, K_{k,pub,t-1}$$

After the planning horizon the depreciations for each type of capital, $P^{KDepr\_pub}_{k,t} K_{k,pub,t-1}$, are an exogenous share, $\alpha^{Depr2Y\_pub}_{k,t}$, of public production excluding production taxes[11], $P^{Y0}_{pub,t} Y_{pub,t}$:

$$P^{KDepr\_pub}_{k,t} K_{k,pub,t-1} = \alpha^{Depr2Y\_pub}_{k,t-1} P^{Y0}_{pub,t} Y_{pub,t}$$

With a constant depreciation rate and proportionality between prices of investment, capital and depreciations this implies a constant K/Y-relationship.[12]

Nominal material inputs, $P^{R}_{pub,t} R_{pub,t}$, and labor inputs, $P^{L}_{pub,t} L_{pub,t}$, are given as shares of production:

$$P^{R}_{pub,t} R_{pub,t} = \left(1 - \sum_{k} \alpha^{KY\_pub}_{k,t-1}\right) \alpha^{RLR\_pub}_{t} P^{Y0}_{pub,t} Y_{pub,t}$$

$$P^{L}_{pub,t} L_{pub,t} = \left(1 - \sum_{k} \alpha^{KY\_pub}_{k,t-1}\right) \left(1 - \alpha^{RLR\_pub}_{t}\right) P^{Y0}_{pub,t} Y_{pub,t}$$

where $\alpha^{RLR\_pub}_{t}$ is materials share and $\left(1 - \alpha^{RLR\_pub}_{t}\right)$ is labors share after depreciations are subtracted.

[11] It is assumed that the public sectors optimal input allocation is unaffected by production taxes - as it pays the taxes to itself. The taxes are only added in determining the production price.

[12] We prefer having an exogenous rate between depreciations and production because the depreciations in the national account is used as the public cost of capital. This is seen as nominal public production are depreciations plus wage costs plus material costs. In this sense it works as a usercost for the public sector. In a Cobb-Douglas production function this gives a constant share of depreciations to production and the above formulation is more directly theoretically consistent with a Cobb-Douglas production function.

4.3 The price index of public production

The price of public production is the cost of public production per unit of production. The costs of public production equals the costs of materials, capital and labor including production taxes. With the input method the value of the production equals the costs. The production value is given by material costs, wage costs, capital depreciations and production taxes.

The price for materials in the public sector, $P^{R}_{pub,t}$, is determined exactly as for the private sectors and is modeled in the Input/Output section.

For private sectors the price of capital is given by usercost. With the input method the cost of capital equals depreciations and the depreciation costs de facto becomes the usercost of capital. The cost of depreciations relative to the capital stock, $P^{KDepr\_pub}_{k,t}$ is given as the investment price of the relevant type of capital, $P^{I}_{k,t}$, times the depreciation rate, $\delta_{k,pub,t}$, times a multiplicative correction term, $\lambda^{pKDepr\_pub}_{k,t}$:

$$P^{KDepr\_pub}_{k,t} = \lambda^{pKDepr\_pub}_{k,t} \delta_{k,pub,t} P^{I}_{k,t}$$

The correction term is necessary to match the depreciation data as public depreciations and public investments are composite goods and have different price indices in the national account. With chain indices this implies that the difference in the relative prices is caught in our depreciation rate and we need a correction term to determine the nominal cost of depreciations.

In the public sector workers must receive the same wage for one effective unit of labor as in all other sectors. Sector specific wage sum taxes and wage subsidies can change this, but apart from this the cost of one unit of effective labor, $P^{L}_{pub,t}$, must follow the wage for one effective unit of labor, w_t[13]:

$$p^{L}_{pub,t} = w_t$$

The average wage per worker, $(w_t L_{s,t}) / n^{L}_{s,t}$, in the different sectors can vary even though the wage per effective unit of labor, w_t, is the same. Some sectors simply employ people who supply more effective units of labor than others. Our model does not explain why sectors differ. Employment in the public sector, $n^{L}_{pub,t}$, is as explained in the firm chapter given by:

$$n^{L}_{pub,t} = \frac{\lambda^{nL}_{pub,t} L_{pub,t}}{\sum_s \lambda^{nL}_{s,t} L_{s,t}} \left(n^{Employed}_{t} + n^{CrossBorder}_{t} \right)$$

where $\left(n^{Employed}_{t} + n^{CrossBorder}_{t} \right)$ is total employment in heads from both households and foreign workers, $L_{s,t}$, is labor input in efficient units in sector s, and $\lambda^{nL}_{s,t}$ is a correction factor to calibrate to sector specific employment. As $P^{L}_{pub,t}$ increases over time $L_{pub,t}$ will decrease relative to the other inputs and this implies that the average wage in the public sector is given by $\frac{\sum_s \lambda^{nL}_{s,t} L_{s,t}}{\lambda^{nL}_{pub,t} L_t} \frac{w_t L_t}{n^{Employed}_{t} + n^{CrossBorder}_{t}}$. The second bracket is the average wage in the economy as a whole. The first term is a correction factor for the public wages. If sectors with more efficiency units per workers expand, the nominator gives a shift downwards in public wages as there will be a crowding out of workers with high productivity in all other sectors including the public. You can increase the public wage relative to other wages by decreasing the exogenous variable $\lambda^{nL}_{pub,t}$. Wages increase as people employed in the public sector in average supplies more efficiency units of labor.

[13] The public sector does not change its marginal price on labor as a consequence of wage sum taxes as it is a tax paid to the public sector itself.

This will decrease wages per worker in other sectors and the average wage per worker in the economy as a whole will be unchanged. It will also decrease the number of people employed in the public sector as they are now more productive and the wage cost in the public sector will be unchanged.

The price index for public production excluding production taxes is a Paasche chain price index combining its three inputs:

$$P^{Y0}_{pub,t} = P^{Y0}_{pub,t-1} \frac{\sum_k P^{KDepr\_pub}_{k,t} K_{k,pub,t-1} + P^{R}_{pub,t} R_{pub,t} + P^{L}_{pub,t} L_{pub,t}}{\sum_k P^{KDepr\_pub}_{k,t-1} K_{k,pub,t-1} + P^{R}_{pub,t-1} R_{pub,t} + \left(1 + g^{Gov}_t\right) P^{L}_{pub,t-1} L_{pub,t}}$$

where $g^{Gov}_t$is the Harrod neutral productivity growth rate in the public sector. The price indices for the private sectors are written differently, but are in principle given by the same equation times a constant mark-up. On a balanced growth path all prices (and hence usercost and the price of depreciations) grow with the inflation rate and wages increase with the inflation rate plus the Harrod neutral growth rate. If the public sector has the same growth rate as the private sector the price index for public consumption will also increase with the inflation rate. It is, however, assumed that there is no productivity growth in the public sector. This implies that its price index will grow with a higher rate. With a constant cost share to labor this will be a constant higher growth rate i.e. neither accelerating or decelerating.

The price index for public production including production taxes, $P^{Y}_{pub,t}$, are calculated in the pricing module.

4.4 Calibrating public production

The calibration of the public production is mostly static. The parameters $\mu^{PublicDirectInvestment2GVA}_t$, $\mu^{PublicNewRealInvestment}_t$, $\lambda^{PublicNewRealInvestment}_t$ and $\alpha^{RLR\_pub}_t$ are calibrated in order for $vPublicDirectInvestment_t$, $vPublicNewRealInvestment_t$, $P^{PublicNewRealInvestment}_t$ and $R_{pub,t}$ to fit the data. The rate of depreciations relative to production, $\alpha^{Depr2Y\_pub}_{k,t}$, needs to be dynamically calibrated as it sets the depreciations and hence the level of capital relative to production in the next period. It is calibrated to give the correct level of investments, $I_{i,pub,t}$. Given the production and the inputs to production including costs of labor, $P^{L}_{pub,t} L_{pub,t}$, it is possible to calibrate the implicit Harrod neutral growth rate for labor, g^{Gov}_t. In all the above equations the calibration is simply to invert the above equations isolating the parameter to be calibrated. This is in the code done automatically by exogenizing the data covered variable and endogenizing the parameter to be calibrated.

5 Sector prices

The sector prices before indirect taxes for the private sectors, $P^Y_{s,t}$, are determined in this section. The price index for production in the public sector is described in the chapter concerning the public production. The prices on the demand components are weighted import and sector prices with duties, VAT and customs. This is described in the Input/Output chapter. The prices on demand components determine the composition of material inputs, consumption, and investments as well as the level of imports and exports through substitution to other demand components and foreign competition. To determine the sector prices only sector specific marginal production costs, $P^{Y0}_{s,t}$, are needed.

The marginal cost of production is determined in the chapter concerning the firm and its production inputs. In perfect competition firms would set their prices to match marginal costs. In a basic set up with monopolistic competition and no price rigidity, wages would be set as a mark up over marginal costs. In MAKRO we have both monopolistic competition and price rigidity. The price rigidity is modeled with Calvo-pricing[14].

5.1 Calvo pricing

Within each sector firms are subject to monopolistic competition. In the monopolistic consumption set-up all firms within each sector faces the same demand elasticity, $\sigma^{YY}_{s,t}$, and the aggregated price over all firms in a given sector, $P^Y_{s,t}$, is a CES price index. If prices were flexible this would give the prices as a markup, $markup_{s,t}$, times the marginal cost of production, $P^{Y0}_{s,t}$. Prices are sticky and only a share of the firms, $1-\mu^{pYrigidity}$, can reset prices each period. The other prices from the other firms automatically increase with an exogenous rate π. All firms are identical, so all firms who can set prices this period set the same price, $P^{Ys}_{s,t}$, and this is weighted with the auto-increased price from the previous period:

$$P^Y_{s,t} = \left[\mu^{pYrigidity}\left((1+\pi)\,P^Y_{s,t-1}\right)^{1-\sigma^{YY}_{s,t}} + \left(1-\mu^{pYrigidity}\right)\left(P^{Ys}_{s,t}\right)^{1-\sigma^{YY}_{s,t}}\right]^{\frac{1}{1-\sigma^{YY}_{s,t}}}$$

Note that the equation in the code is written without the $(1+\pi)$ since it is adjusted for growth and inflation. Growth adjustment is explained in an appendix.

The firms who reset prices set the optimal price given that each period prices can only be reset with a certain possibility and given the demand elasticity facing their product:

$$P^{Ys}_{s,t} \quad = \quad (1+\chi_{s,t})\,\frac{numerP_{s,t}}{denomP_{s,t}}$$

$$numerP_{s,t} = Y_{s,t}P^{Y0}_{s,t}\left(P^Y_{s,t}\right)^{\sigma^{YY}_{s,t}} + numerP_{s,t+1}\frac{\mu^{pYrigidity}_{s,t}}{1+r^{firms}_{t+1}}$$

$$denomP_{s,t} = Y_{s,t}\left(P^Y_{s,t}\right)^{\sigma^{YY}_{s,t}} + denomP_{s,t+1}\frac{\mu^{pYrigidity}_{s,t}}{1+r^{firms}_{t+1}}$$

The above expression is derived in Appendix **??**, Deriving Calvo pricing. The parameter $\mu^{pYrigidity}_{s,t}$ is reserved to match impulse-response functions. $\chi_{s,t}$ is calibrated to set

[14] The price setter disregards effects from prices to the capital stock to marginal costs via quadratic installation costs. This is a bit strange if the price setter is also the capital goods producer. To accept this assumption imagine every sector has two sub-sectors: an intermediate sector with perfect competition producing output and a wholesale sub sector with monopolistic competition buying at marginal costs and setting the price with rigidity.

the mark up, and $\sigma_{s,t}^{YY}$ is set to 0. The mark up should equal $\frac{\sigma_{s,t}^{YY}}{\sigma_{s,t}^{YY}-1}$, but as the mark up in some sectors in some years is calibrated to be negative, this gives problems and, as a fix, it is set to 0 in this model version.

5.2 Data and calibration

Given the Calvo parameter sectoral prices are fitted to the national accounts data from ADAM's databank by dynamically calibrating the Calvo markups. As prices today depend on future prices, the calibrated Calvo markups today depend on future Calvo markups. The future markups are equal to the markup in the last period.

6 Labor market

A model of the labor market must be able to reproduce the level and behavior of wages, unemployment rates, and labor force participation. Of particular importance is the ability of the model to reproduce how much of the cyclical adjustment falls on employment versus on wages. In addition, objects such as unemployment rates and participation rates will vary over the life cycle, and we want our model to be able to also match this age variation.

The modern benchmark model of the labor market in macroeconomics is the search and matching model, and a good overview of its properties can be found in Ljungqvist and Sargent (2016). We are currently developing such a model, so in the meantime we use a Phillips Curve model which we detail here and which covers some of the facts we aim to explain. This model is taken from A. Mortensen (2015). The Phillips curve model is part of what we call supply and demand models while the search and matching model departs somewhat from this way of looking at the market. A recent review and debate of the Phillips Curve in macroeconomics can be found in Gordon (2013) and Coibion and Gorodnichenko (2013).

6.1 Supply and Demand

In order to provide intuition about how the Phillips Curve model works it is useful to look at a very simple supply and demand model of the labor market. Consider a downward-sloping labor demand given by the marginal product of labor curve (MPL), $p\frac{\partial F(L)}{\partial L} = w$. All models of the labor market share this feature, although the exact expression of the marginal product of labor relationship will vary. Close the market with a vertical line (the exogenous labor force) and flexible wages. Equilibrium with positive wages is then found on the vertical labor force line. There is of course no unemployment.

In order to obtain unemployment we need a labor supply. Early Real Business Cycle models added disutility of work in the utility function as follows: the household likes consumption and leisure $(T - L)$ and maximizes $U(C) + g(T - L)$ with g concave. The first order condition

$$\frac{\partial U}{\partial C}\frac{w}{p} = \frac{\partial g(T-L)}{\partial (T-L)}$$

results in an upward sloping curve. In this version, the total time endowment T stands for the vertical labor force and the quantity $T - L$ is the equivalent of labor resources not used. However, it is not unemployment as everyone that wants to work is working.

We still lack a way of introducing unemployment in its real sense, where people want to work but cannot find a job. This is where the wage Phillips curve comes in and replaces the leisure choice above. In this example we write

$$w_t = w_0 \times exp\left\{-\psi(u_t - u_0)\right\} \equiv w_0 \times exp\left\{-\psi\left(\frac{T-L_t}{T} - u_0\right)\right\}$$

where in the right hand side identity we replace $u_t = (T - L_t)/T$, and where equilibrium unemployment is found relative to an exogenous target unemployment rate u_0. As the Phillips curve assumes a negative relationship between wages and unemployment, it necessarily implies a *positive* relationship between wages and *employment*, and therefore plays the same role as the previous disutility curve. We can now turn to the Phillips Curve model we actually use.

6.2 Definitions

The following are basic elements of the wider model which are useful to understand the labor market discussed here. A household encompasses all living agents of a given age

$N_{a,t}$. Household members can be in one of four labor market states: employed $\left(n_{a,t}^{Emp}\right)$, unemployed $\left(n_{a,t}^{Unemp}\right)$, out of the labor force $\left(n_{a,t}^{Olf}\right)$, and retired $\left(n_{a,t}^{Ret}\right)$. In the current model version the labor force is exogenous, both $n_{a,t}^{Olf}$ and $n_{a,t}^{Ret}$ are exogenous. In later versions with endogenous labor supply, only $n_{a,t}^{Ret}$ is exogenous.

Finally, we define efficiency units of labor as the product of the number of workers times a productivity factor ρ.

$$L = \rho n^{Emp}$$

6.3 The Phillips curve model

Labor demand is the marginal product of labor curve. The firm hires divisible productive units of labor, L, rather than persons, n. In the simplest version of the model each firm j solves the following static labor choice problem:

$$V_t^j = max_{L_t^j} \left\{ p_t^j F\left(L_t^j\right) - w_t L_t^j + \tilde{\beta}_{t+1} V_{t+1}^j \right\}$$

The first order condition for this problem, and the demand curve, is

$$p_t^j \frac{\partial F\left(L_t^j\right)}{\partial L_t^j} = w_t$$

This problem is of course extremely simple as in many models labor is a quasi-fixed factor. As an example, adjustment costs to labor can be added to the demand side of the model as a separate block without any effect on the rest of the algebra in the model.

On the supply side wages are set via the Phillips curve.

$$\frac{w_t}{w_{t-1}} = \frac{w_{t+1}}{w_t} x_t^0 \left\{ \frac{1}{2} e^{-\psi(u_t - u_t^0)} + \frac{1}{2} e^{-\psi(u_{t-1} - u_{t-1}^0)} \right\}$$

where u_t^0 is an exogenous variable and where x_t^0 is an exogenous growth factor. The timing of the unemployment gap is contemporaneous which makes the model close to the static labor supply curve discussed in section 2. It acts as an upward sloping labor supply curve with a position conditional on past and future wages. Here, an unemployment rate lower than the target is associated with current wage inflation higher than expected future wage inflation. How strong this effect is depends on the slope parameter ψ, as the Phillips Curve introduces not only unemployment but also wage rigidity.

As the demand for labor is defined in efficiency units and the supply is defined in number of workers, we must define a few objects by age in order to close the model. Age-specific unemployment rates are given by $u_{a,t}^a = \xi_t u_{a,t}^0$, and where ξ_t is a key adjustment variable. Employment is given by $n_{a,t}^{Emp} = LF_{a,t}(1 - u_{a,t}^a)$ where $LF_{a,t}$ is the exogenous labor force in persons obtained from the data, and unemployment is then $n_{a,t}^{Unemp} = LF_{a,t} u_{a,t}^a$, while the aggregate unemployment rate is

$$u_t = \frac{\sum_a LF_{a,t} u_{a,t}^a}{\sum_a LF_{a,t}}$$

We are now ready to define the market clearing relationship

$$L_t^d\left(w_t\right) = \sum_j L_t^{d,j}\left(w_t\right) = \sum_a \rho_{a,t} n_{a,t}^{Emp}$$

where L_t^d is the total labor demand in efficiency units from all firms. This condition also highlights the fact that, even though workers of different ages may have different productivity levels, the firm does not care who it hires. The market wage is the price of one productive unit of labor.

Finally, since ξ is not age dependent unemployment across all ages moves the same way, a case of “the tide raising all boats by the same factor”. By construction the age distribution of employment and unemployment is always the same as that found in the data up to a proportionality factor.

6.4 Conclusion

This model allows us to generate unemployment, and contains a degree of wage rigidity, allowing for a decomposition of the adjustment over the cycle between wages and unemployment. It also generates age distributions of unemployment and employment which are restricted to move synchronously with the aggregate unemployment rate.

A final word is in order with respect to the search/matching/bargaining model. In this model the search component and the value of a filled job typically provide the surplus from a firm-worker match which is to be divided through the solution of the bargaining problem. While the expression of the marginal product of labor curve may now be extended to include vacancy posting costs, it is nevertheless the ever present downward sloping demand curve. What then replaces the Phillips curve or the labor/leisure choice is the first order condition of the solution to the bargaining problem. The properties of this first order condition are endogenous and due to the non-linearity of the problem they are best compared to the Phillips curve or the neoclassical model through computational methods.

7 Households

Our benchmark model is a discrete time, perfect foresight, cohort-based, life cycle one, and in it a household encompasses all living agents of a given age. The timing convention is that all decisions are taken, income is realized, and consumption occurs at the end of each period. At that point cohort members share their income and any assets carried over from the previous period. The size of the cohort aged a in period t is given by $N_{a,t}$ and this quantity is exogenous and obtained from the data.

The household derives utility from consuming non durable goods.[15] The household problem is to choose an optimal consumption and savings path over the life cycle given its income path. With a frictionless access to finance the solution to this problem yields the usual consumption smoothing outcome which comes out of the intertemporal first order condition. Consumption follows a flat trend over the life cycle thereby smoothing a hump-shaped income path. The data, however, paints a different picture, with consumption tracking income, and the key element that allows our model to match the data is the departure from frictionless finance.[16] We introduce household banking to answer the following question: "why does consumption track income?". The simplest answer to this question is that consumption tracks income because it must be expensive to borrow *and* expensive to save. And given this answer we choose to model directly the cost of saving and borrowing. The model then allows us to generate the observed gap between the rate of return on savings in the bank and the interest cost of borrowing from the bank. Costly finance also impacts the dynamic response of consumption and savings to policy shocks.[17]

A popular exogenous way to handle the property that consumption follows income over the life cycle is to assume a fraction of rule of thumb agents in the spirit of Campbell and Mankiw (1989). These agents do not save but rather consume their entire disposable income every period. Here any policy change that impacts disposable income will have a one to one response of consumption for these agents, and in this way impact the response of aggregate consumption directly. Again, modeling the borrowing and lending channel explicitly is a substitute for this ad-hoc device. Nevertheless, in order for our model to be comparable with other work that uses such a device we allow for a fraction ξ of the population to be rule of thumb consumers.

7.1 Utility

Households have a discount rate θ. Instantaneous utility is the linear sum of individual utilities of all living agents of a given age:

$$U_{a,t} \equiv (1-\xi)N_{a,t}\frac{1}{1-\sigma}\left[\tilde{C}_{a,t}\right]^{1-\sigma} + \xi N_{a,t}\frac{1}{1-\sigma}\left[\frac{y_{a,t}}{P_t^C}\right]^{1-\sigma}$$

Here $\tilde{C}_{a,t} = \hat{C}_{a,t} - hC_{a,t-1}$ denotes consumption net of habitual consumption where the latter is of degree h. The agent compares himself with the average agent of the same age in the previous period, but crucially not with itself. The consumption of rational agents $\hat{C}_{a,t}$ and total cohort consumption, $C_{a,t}$, are related through

[15] The household decides also on its participation in the labor force, but that choice is discussed in the labor market chapter. Furthermore, consumption is the result of a CES nest optimization sequence which relates to the input-output structure of the data. This is also detailed elsewhere.

[16] Browning and Ejrnæs (2009) show that after removing expenditure with children, the remaining consumption expenditure is flat over the life cycle.

[17] Boldrin, Christiano, and Fisher (2001) identify a low elasticity of the supply of capital as a key element that helps match the Equity premium puzzle. Our banks should be able to deliver such low elasticity.

$$C_{a,t} = (1-\xi)\hat{C}_{a,t} + \xi \frac{y_{a,t}}{P_t^C}$$

In the data disposable income includes interest income so that the variable $y_{a,t}$ is labeled net primary income. The optimizing fraction of the living cohort of age a at time t maximizes the following discounted sum

$$\sum_{s=0}^{A-a} \left\{ (1-\xi)N_{a+s,t+s} \left[\frac{\tilde{C}_{a+s,t+s}^{1-\sigma}}{1-\sigma} \right] \left(\frac{1}{1+\theta} \right)^s \right\}$$

where for simplicity we use a constant discount rate, and where A is the last age of life.

7.2 Budget constraint

The budget constraint of non rule of thumb agents is

$$\hat{B}_{a,t} = [1 + r_{a-1,t-1}] \times \hat{B}_{a-1,t-1} \times \frac{(1-\xi)N_{a-1,t-1}}{(1-\xi)N_{a,t}} + y_{a,t} - P_t^C \hat{C}_{a,t}$$

$$\hat{B}_{A,t} = 0, \quad \hat{B}_{0,t} = 0$$

where A is the final age and zero denotes beginning of period at age 1. The agent starts life with zero assets and ends life the same way. Imposing zero assets at death implies we automatically satisfy the no-ponzi condition. Assets $\hat{B}_{a,t}$ are defined as end-of-period, as we label them relative to the age and period where they have been determined.

In the budget constraint we observe the pooling of assets characteristic of the cohort model as total carried-over assets, $\hat{B}_{a-1,t-1} \times (1-\xi)\, N_{a-1,t-1}$, are divided equally among the optimizing members of the current cohort, $(1-\xi)\, N_{a,t}$. Rule of thumb agents are excluded from sharing. We can also see that the rate of return on assets has an age index. This is due to the fact that this rate of return is itself a function of assets which in turn are indexed by age.

7.3 First order conditions

The dynamic first order conditions can be obtained mechanically by replacing the consumption variable with the budget constraint in the sequence problem, and choosing end of period assets at every age. The intuition behind it is as follows. The optimizing fraction of the cohort aged a at time t obtains, from the last unit of income used for current consumption, the marginal utility of consumption evaluated at current prices P_t^C:

$$(1-\xi)N_{a,t}\tilde{C}_{a,t}^{-\sigma} \frac{1}{P_t^C}$$

Optimality implies this must be identical to what the cohort obtains from alternatively saving this marginal unit of income, earning a gross return $R_{a,t}$, and using it next period for consumption, taking into account that this additional income is shared among next period's optimizing cohort members, $(1-\xi)\, N_{a+1,t+1}$. This is given by

$$\left\{ (1-\xi)N_{a+1,t+1}\tilde{C}_{a+1,t+1}^{-\sigma} \frac{1}{P_{t+1}^C} \right\} \frac{R_{a,t}(1-\xi)N_{a,t}}{(1-\xi)N_{a+1,t+1}}$$

and must be discounted by the factor $\frac{1}{1+\theta}$ to match the current marginal utility. As the cohort size cancels out from both sides we obtain:

$$\frac{1}{P_t^C} \times \tilde{C}_{a,t}^{-\sigma} = \frac{1}{P_{t+1}^C} \times \tilde{C}_{a+1,t+1}^{-\sigma} \left(\frac{R_{a,t}}{1+\theta} \right)$$

where

$$R_{a,t} = 1 + r_{a,t} + \hat{B}_{a,t}\frac{\partial r_{a,t}}{\partial \hat{B}_{a,t}}$$

These conditions determine the shape of the path of consumption independently of income. However, as the rate of return is a function of age due to portfolio considerations, the consumption path will have a life cycle profile.

7.4 The rate of return on assets

Households can place their liquid wealth in stocks and bonds or in the bank. The rate of return on portfolio assets has an exogenous composition over bonds and stocks, ω^b, and over domestic and foreign assets, ω^f. Domestic and foreign bonds are assumed to be identical. There is also an exogenous partition into what is invested in portfolio assets and what is deposited in the bank, ω^{banks}. The rate of return $r_{a,t}$ associated with assets $\hat{B}_{a,t}$ is then given by

$$\begin{gathered} r_{a,t} = \left(1-\omega^{banks}_{a,t}\right) \times \omega^b_{a,t} \times r_{t+1} \\ + \\ \left(1-\omega^{banks}_{a,t}\right) \times \left(1-\omega^b_{a,t}\right) \times \omega^f_t \times \left[\frac{V^F_{t+1}}{V^F_t} - 1 + \frac{DIV^F_{t+1}}{V^F_t}\right] \\ + \\ \left(1-\omega^{banks}_{a,t}\right) \times \left(1-\omega^b_{a,t}\right) \times \left(1-\omega^f_t\right) \times \left[\frac{V^D_{t+1}}{V^D_t} - 1 + \frac{DIV^D_{t+1}}{V^D_t}\right] \\ + \\ \omega^{banks}_{a,t} \times \left\{r_{t+1} - \lambda^{bank}\left[\hat{B}_{a,t}\right]\right\} \end{gathered}$$

Where V is the value of the firm and DIV are dividends, and r_t is not only the money market rate but also the world bond rate. Note that the portfolio weights $\omega_{a,t}$ are timed consistently with $\hat{B}_{a,t}$, as the portfolio decision pertains to the assets invested. On the other hand, the bond rate r is timed $t+1$ instead of t because, just as the return on stocks is realized ex-post, bond contracts do not necessarily have ex-ante defined rates of return.[18]

Here $\omega^b_{a,t}$ includes a dummy variable $PortD_a$ to make sure that, at young ages when assets are negative, households do not dissave into stocks as they would effectively be issuing them. Thus, $PortD_a = 0$ at young ages:[19]

$$\omega^b_{a,t} = \omega^b_t \times PortD_a + 1 - PortD_a$$

Regarding the effective interest rate on deposits, since $\lambda_{bank} > 0$ when $\hat{B} > 0$, the interest rate received by households is smaller than the rate which the bank gets in the money market, r_t, and vice versa. The portfolio contains foreign assets. These are assumed to behave identically to domestic assets in the normal run of the model. However, when we shock the model we assume foreign assets do not respond and their time series remains identical to the non shock run of the model.

7.5 Banks

Banks are, for the moment, neutralized by imposing zero profits so they disappear from the model. Banks trade with the world money market at the bond rate r. They in turn borrow from and lend to households. They profit from both types of transactions, as they

[18] A bond contract is defined by priority of claim rather than by the rate agreed upon.

[19] We also set the value of $w^{banks}_{a,t}$ so that at young ages agents have only the bank as a savings vehicle.

earn $r_t - r^h_{a,t}$ on their deposits, and $r^h_{a,t} - r_t$ on their loans. Their profits are then given by

$$\pi^{Bank}_t = \lambda^{bank} \sum_a \left[\omega^{banks}_{a-1,t-1} \times \hat{B}^2_{a-1,t-1} \times (1-\xi) N_{a-1,t-1} \right] - \kappa^{bank}_t$$

and we allow for ad-hoc bank operational costs κ^{bank}_t to vary with time in order to keep profits at zero with a constant household banking cost lambda. These costs are modeled as resources sent abroad permanently.

We set the pair $\left(\lambda^{bank}, \kappa^{bank}_t\right)$ to either zero, to shut down the banking channel, or, when positive, to match the active/passive interest rate dispersion observed in the data. For example, the June 2015 interest rate on bank deposits by non financial corporations was 0.227% and for deposits by households was 0.443%. On the other hand, the interested rate charged on bank loans to households was 3.569% for housing and 5.605% for non housing.[20]

7.6 Growth and inflation corrections

Reconciling a stationary model with the data requires correcting for exogenous average growth and inflation rates. The appendix explains in detail how this is done. For the current problem we obtain the following transformed budget constraint

$$\hat{B}_{a,t} = [1 + r_{a-1,t-1}] \times \hat{B}_{a-1,t-1} \times \frac{(1-\xi) N_{a-1,t-1}}{(1-\xi) N_{a,t}} \left[\frac{1}{1+g_t} \frac{1}{1+\pi_t} \right] + y_{a,t} - P^C_t \hat{C}_{a,t}$$

we obtain the first order condition

$$\tilde{C}_{a+1,t+1} \left(1 + g_{t+1}\right) = \left(\frac{P^C_t}{P^C_{t+1}} \frac{1}{1+\pi_{t+1}} \times \frac{1 + r_{a,t} + \hat{B}_{a,t} \frac{\partial r_{a,t}}{\partial \hat{B}_{a,t}}}{1+\theta} \right)^{\frac{1}{\sigma}} \tilde{C}_{a,t}$$

These equations are written with time varying correction factors π_t and g_t for ease of reference with the appendix and ease of calculation. However, in our implementation these factors are constant as what is effectively being done is to ensure stationarity of the model relative to a long run path in the actual data.

7.7 Final Note

This model forms the basic structure of the household problem. It has a single asset and a single consumption good, and contains one innovation relative to its precursor, the DREAM model, which is the banking channel. Broader versions of this problem include, among other features, housing, mortgages, and bequests. In these more general models the specific model of the banking relationship is developed to replicate finer properties of the financial position of the household.

[20] The loan rate on housing mortgage bonds was, however, only 1.084%.

8 Consumption components

In this section the consumption components, $C_{c,t}$, are determined. These are used in the Input/Output chapter to find out how the demand from households translates into demand on specific domestic sectors and import groups. The price indices for the different consumption components are determined in the Input/Output section as a weighted average of the prices from inputs from domestic production and imports. As a general rule in the CES nests: 1) prices and quantities are determined simultaneously, 2) quantities are modeled top down, and 3) prices are modeled bottom up.

Aggregate private consumption is given as the aggregate consumption from all cohorts in a given year[21]: $C_{tot,t} = \sum_a C_{a,t} N_{a,t}$. The total private consumption is divided into consumption groups according to the structure of the nested CES-tree to give the demand for different consumption groups, $C_{c,t}$, that maximizes utility:

$$C_{c\_,t} = \mu^C_{c\_,t} C_{cu,t} \left(\frac{P^C_{c\_,t}}{P^C_{cu,t}} \right)^{\sigma^C_{cu}}$$

where the set $c\_$ consists of all the components from the sets c and cu. The set cu is a set with all upper nests in the consumption tree. In general μ denotes a share parameter and σ denotes an elasticity. The superscript C indicates that it is for the consumption part of the model. This one equation is used for all nests and has a substitution elasticity for each nest i.e. for each cu.

There are 6 lower consumption groups: cars, energy, goods, housing, services and tourism, $c = cCar, cEne, cGoo, cHou, cSer, cTou$. In this model version there is only one upper nest with $cu = tot$. So in this specific model version the above equation has only one nest and six equations:

$$C_{c,t} = \mu^C_{c,t} C_t \left(\frac{P^C_{c,t}}{P^C_t} \right)^{\sigma^C}$$

In a later model version we could have more nests for example non-housing. So $cu = CNON, tot$ and instead of representing only the equations above the compressed equation expresses the above and the following two equations:

$$C_{CHOU,t} = \mu^C_{CHOU,t} C_{tot,t} \left(\frac{P^C_{CHOU,t}}{P^C_{tot,t}} \right)^{\sigma^C_{tot}}$$

$$C_{CNON,t} = \mu^C_{CNON,t} C_{tot,t} \left(\frac{P^C_{CNON,t}}{P^C_{tot,t}} \right)^{\sigma^C_{tot}}$$

$$C_{c\_non,t} = \mu^C_{c\_non,t} C_{CNON,t} \left(\frac{P^C_{c\_non,t}}{P^C_{CNON,t}} \right)^{\sigma^C_{CNON}}$$

where $c\_non = cCar, cEne, cGoo, cSer, cTou$.

There are both imports and exports of tourism. Imports of tourism are how much Danish households consume abroad and are given by $C_{cTou,t}$ above. It is a consumption good and increases with national wealth. Export of tourism are determined in the foreign sector chapter and its aggregate is given by $X_{xTou,t}$. Foreign households tourist consumption in Denmark is also divided on consumption groups in the foreign sector

[21] Assuming all cohorts have the same consumption decomposition and hence the same price index for consumption allows us to use a simple sum in this identity.

chapter and is given by $C_{c,t}^{Tourist}$. The Danish consumption groups are given by the aggregate consumption of Danish households and tourists i.e. by $C_{c,t} + C_{c,t}^{Tourist}$. It is in the model assumed that tourists and Danish households face the same price for the same consumption components. This is not necessarily the case in data and it is captured in the relationship between $X_{xTou,t}$ and $\sum_c C_{c,t}^{Tourist}$.

Parameters are statically calibrated to match national accounts consumption groups from ADAM's databank subtracted tourist consumption.

9 Foreign sector

The foreign sector is mostly an integrated part of the Input/Output system and is described in this chapter. The import from a specific import sector is determined as the sum of deliveries from this import sector to the different demand components. The exports are divided into export components consisting of energy, goods, services and tourism referred to with a suffix x=xEne,xGoo,xSea,xSer,xTou. The export components include both direct exports and import to reexports are described below given by an Armington model. In the Input/Output system export components are divided into delivering sectors and further into domestic deliveries and import to reexport. The export price indices are taken from the Input/Output system.

Exports, $X_{x,t}$, are given according to an Armington model for each separate export component:

$$X_{x,t} = \lambda_t^X X_{x,t-1}(1+g_t) + \left(1-\lambda_t^X\right) \mu_{x,t}^X MF_{x,t} \left(\frac{P_{x,t}^X}{P_{x,t}^{XF}}\right)^{\sigma_x^X}$$

where λ_t^X (called $Xrigidity_t$ in the code) is here to give an ad hoc gradual response to price changes (in this version it is independent on sector),$\mu_{x,t}^X$ is a share parameter, $MF_{x,t}$ is export market size taken from ADAM, the relative prices are the sectors export price, $P_{x,t}^X$, relative to its export competing price, $P_{x,t}^{FX}$. $(1+g_t)$ is included so that the rigidity does not affect the steady state path. $P_{x,t}^{XF}$ is the competing foreign prices of the export market taken from ADAM. The export elasticity, σ_s^X, is as in DREAM set to 5 for all sectors.

One of the export components are exports i.e. the expenditures of foreign tourists in Denmark. Their total consumption is determined in the equation above and then it is divided into consumption components with the following equations:

$$C_{c,t}^{Tourist} = \mu_{c,t}^{CTourist} X_{XTOU,t}$$

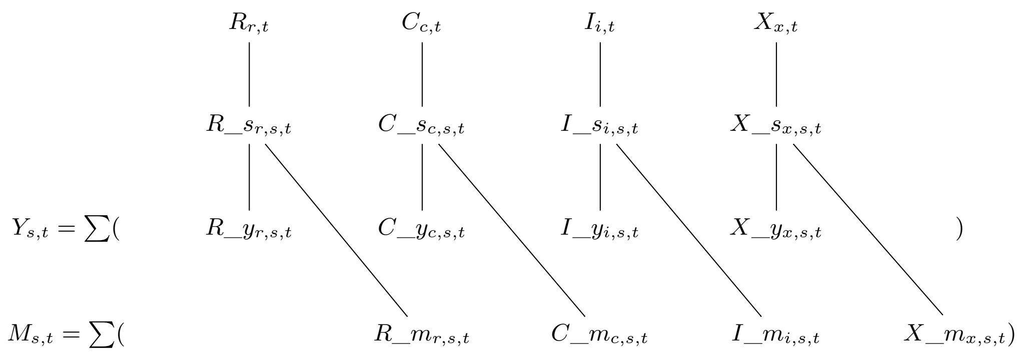
The weights $\mu_{c,t}^{CTourist}$ are balanced as described in the IO chapter.

Parameters are statically calibrated to match national accounts export groups from ADAMs databank and tourist consumption. Tourist consumption divided on consumption groups is not official national accounts data. It is imputed by keys from ADAMs equation for the price index of tourist exports giving fixed weights to different consumption groups. The parameter $\mu_{c,t}^{CTourist}$ is, however, not constant as it captures the difference between the actual price of tourist exports and a simple weighted average price of its assumed components in MAKRO.

10 The Input/Output system

The core of the model is the Input/Output system. It connects the demand and supply of goods and services. The demand components are material inputs to production, $R_{r,t}$, private consumption, $C_{c,t}$, government consumption, $G_{g,t}$, investments, $I_{i,t}$ and exports, $X_{x,t}$. The demand is satisfied with input from domestic production, $Y_{s,t}$, and imports, $M_{s,t}$. How the demand components are determined is described in the other chapters.

You can think of each demand component in, $R_{s,t}$, $C_{c,t}$, $G_{g,t}$ and $I_{i,t}$, as being created in a final goods sectors with a nested CES production function that operate under perfect competition and hence of the Input/Output system as a description of these final goods sectors. Each demand component is produced with inputs of domestic production and imports from all sectors. Inputs from the domestic sector to the demand components are called, $R\_y_{r,s,t}$, $C\_y_{c,s,t}$, $G\_y_{g,s,t}$, $I\_y_{i,s,t}$ and $X\_y_{x,s,t}$, and inputs imported from sectors abroad are called, $R\_m_{r,s,t}$, $C\_m_{c,s,t}$, $G\_m_{g,s,t}$, $I\_m_{i,s,t}$ and $X\_m_{x,s,t}$. Table 1 shows the demand side of the Input/Output table where the demand components are vertical vectors and the supply components are horizontal vectors.



10.1 The Input/Output equations

The aggregate of domestic production and imports for a specific sector, s, are called $R\_s_{r,s,t}$, $C\_s_{c,s,t}$, $G\_s_{g,s,t}$, $I\_s_{i,s,t}$ and $X\_s_{x,s,t}$. They are given as the solution to the final goods sectors optimization problem. Technical details are given in the appendix on solving nested CES functions. The equations are for $D = R, G, I, X$ and $d = r, g, i, x$ given by:

$$D\_s_{d,s,t} = \mu^{D\_s}_{d,s,t} D_{d,t} \left(\frac{P^{D\_s}_{d,s,t}}{P^{D}_{d,t}} \right)^{-\sigma^D_d}$$

where $\mu^{D}_{d,s,t}$ are calibrated parameters. Typically, the elasticities of substitution, σ^D_d, are set to 0 between these inputs. This gives constant Input/Output coefficients for the aggregate of imports and domestic production for each sector (here called the sector-product) while there is substitution between imports and domestic production for each sector-product. Sectoral inputs to the consumption groups are modeled slightly different as $C_{c,t}$ only represents Danish households consumption where as the input components $C\_s_{c,s,t}$ includes input to consumption from tourists $CTourist_{c,t}$:

$$C\_s_{c,s,t} = \mu^{C\_s}_{c,s,t} \left(C_{c,t} + CTourist_{c,t} \right) \left(\frac{P^{C\_s}_{c,s,t}}{P^{C}_{c,t}} \right)^{-\sigma^C_c}$$

The production from the domestic sectors and import from the corresponding foreign sectors substitute each other in the production with a fixed elasticity of substitution[22]:

$$D\_y_{d,s,t} = \mu^{D\_y}_{d,s,t} D\_s_{d,s,t} \left(\frac{P^{D\_y}_{d,s,t}}{P^{D\_s}_{d,s,t}} \right)^{-\sigma^{D\_s}_{d,s}}$$

$$D\_m_{d,s,t} = \mu^{D\_m}_{d,s,t} D\_s_{d,s,t} \left(\frac{P^{D\_m}_{d,s,t}}{P^{D}_{d,s,t}} \right)^{-\sigma^{D\_s}_{d,s}}$$

where $D = R, C, G, I, X$ and $d = r, c, g, i, x$. In principle the elasticities are allowed to be unique for each demand component. This is an Armington formulation for the import shares for the different demand components. In a later version of the model this formulation can be refined.

10.2 Prices on demand components

The firms in the intermediate sectors set their output prices as described in the price setting chapter. In MAKRO it is assumed that the micro level is the sectoral level. With this assumption all deliveries from a sector has the same price before taxes no matter what demand component it delivers to[23]. The indirect taxes can vary from demand component to demand component. Typically the households face higher indirect taxes on private consumption than firms do on material inputs. The prices on the demand components domestic and imported inputs are given by the price from the delivering sector times an indirect tax rate:

$$P^{D\_y}_{d,s,t} = \left(1 + t^{D\_y}_{d,s,t}\right) P^{Y}_{s,t}$$

$$P^{D\_m}_{d,s,t} = \left(1 + t^{D\_m}_{d,s,t}\right) P^{M}_{s,t}$$

where $D = R, C, G, I, X$ and $d = r, c, g, i, x$.

The indirect tax rates for domestic production are given by:

$$t^{D\_y}_{d,s,t} = \left(1 + tDuty^{D\_y}_{d,s,t}\right)\left(1 + tVAT^{D\_y}_{d,s,t}\right) - 1$$

where $tDuty$ and $tVAT$ are the exogenously given duty and VAT rates taken from the Input-/Output table. They are allowed to vary both across the delivering and receiving sector. In the most dis-aggregated national accounts and in ADAM they are identical for all deliveries s. In MAKRO the variation on this dimension is only due to aggregating more sectors from ADAM. REFORM data exists to make a better split. The indirect taxes for imports are given by:

$$t^{D\_m}_{d,s,t} = \left(1 + tCus^{D\_m}_{d,s,t}\right)\left(1 + tDuty^{D\_m}_{d,s,t}\right)\left(1 + tVAT^{D\_m}_{d,s,t}\right) - 1$$

where $tCus$ are the exogenously given custom rates taken from the Input-/Output table.

The prices on the aggregate supply from both imports and domestic production from each sector to each demand component is given by:

[22] Again this follows from the derivations in the appendix on solving nested CES-functions.

[23] In the Input-Output tables from the national accounts it is possible to derive prices for the different I-O cells. In these cells the net price from each delivering sector will vary. We deliberately disregard this information as to make the model more tractable. The same is done, but less explicitly in the ADAM model. Here no prices are defined for the I-O cells, but using constant I-O coefficients implies the same assumption.

$$P^{D\_s}_{d,s,t} = \frac{P^{D\_y}_{d,s,t} D\_y_{d,s,t} + P^{D\_m}_{d,s,t} D\_m_{d,s,t}}{D\_s_{d,s,t}}$$

Similarly the prices on the demand components are given by:

$$P^{D}_{d,t} = \frac{\sum^{s} P^{D\_s}_{d,s,t} D\_s_{d,s,t}}{D_{d,t}}$$

The prices of both the demand components, $P^{D}_{d,t}$, and supply components, $P^{Y}_{s,t}$ and $P^{M}_{s,t}$, are in the calibrated periods in accordance to the Paasche chain indices from the national account. These indices equals 1 in the base year just as the demand components[24].

10.3 Supply and aggregates

The production of each sector is the sum of deliveries to all demand components for both domestic production and imports:

$$Y_{s,t} = \sum_{r} R\_y_{r,s,t} + \sum_{c} C\_y_{c,s,t} + \sum_{g} G\_y_{g,s,t} + \sum_{i} I\_y_{i,s,t} + X\_y_{x,s,t}$$

$$M_{s,t} = \sum_{r} R\_m_{r,s,t} + \sum_{c} C\_m_{c,s,t} + \sum_{g} G\_m_{g,s,t} + \sum_{i} I\_m_{i,s,t} + X\_m_{x,s,t}$$

Behind this definition lies the assumption that all deliveries from the same sector is the same product and hence has the same price net of taxation.

The aggregate prices of demand components, P^{R}_{t}, P^{G}_{t}, P^{I}_{t} and P^{X}_{t} , supply components, P^{Y}_{t} and P^{M}_{t}, as well as GDP and gross value added has no natural theoretical price index. They are, however, reported in the code as Paasche price indices and their quantities R_t, G_t, I_t, X_t, Y_t, M_t, GDP_t and $GrossValAdd_t$ are calculated as Laspeyres quantity indices:

$$D_t = \frac{\sum^{d} P^{D}_{d,t-1} D_{d,t}}{P^{D}_{t-1}}$$

$$P^{D}_{t} = \frac{\sum^{d} P^{D}_{d,t} D_{d,t}}{D_t}$$

The aggregate for private consumption, C_t, (given as the solution to the households inter-temporal optimization problem) has a natural theoretically price index given by:

$$P^{C}_{t} = \frac{\sum^{c} P^{C}_{c,t} C_{c,t}}{C_t}$$

10.4 Data and calibration

The models Input/Output system in values is a simple aggregation of the Input/Output system of the Danish National Accounts. It can be directly calculated by aggregating up. The data for the current version of the model is, however, based on the data bank from the ADAM-model. ADAM has 12 sectors, 8 private consumption groups, 1 government consumption group, 5 investments groups and 8 export groups. There is a direct mapping

[24]This means that the quantities are indexed at gross prices where the prices from the lower nest are indexed at net prices. This is only a level shift which is caught in the calibrated share parameter. The development in both quantities is net of customs, duties and VAT.

from ADAM's to MAKRO's consumption, investment and export groups. The mapping is cCar=Cb, cEne=Cb+Ce, cGoo=Cf+Cv, cHou=Ch, cSer=Cs, cTou=Ct, for the private consumption groups, G=Co for the public, iM=im, iB=ib, invt=il+ikn+it for investments and xGoo=E01+E2+E59+E7y, xEne=E3, xSea=Ess, xSer=Esq, xTou=Et for exports. We would like to further divide iM into machinery incl. transportation and intellectual rights at some point.

The sectors are almost a precise mapping from ADAM with one exception. The private service sector in MAKRO is defined as all services including public and financial services excluding all public (offentlig forvaltning og service) o1 in ADAM. This gives the mapping agr=a, con=b, ene=ne+ng, ext=e, hou=h, man=nf+nz, sea=qs, ser=qf+qz+o-o1, pub=o1. There is one big advantage and two small disadvantages with this definition. The big advantage is that it defines the public sector with the definition relevant to the ministries. One small disadvantage is that there is a bit of public production in each sector and taking it all from services is only an approximation. Another small disadvantage is that there is no information on the input-structure from and to this definition of the public sector. This is solved by assuming the material inputs to pub is proportional to that of sector o i ADAM and by assuming that all deliveries from the public sector goes to public sales, public direct investments and public consumption. All public sales are assumed to go to private consumption of services and all public direct investments are assumed to go to intellectual rights placed under machinery investments - ie. there are no public exports and no material inputs from the public to the private sectors.

In ADAM it is assumed that investments from all sectors of a certain type has the same deliveries from other sectors. The national accounts has data that gives detailed information about the deliveries to investment types in the different sectors. MAKRO has the same assumption as ADAM mainly in order to reduce the dimensionality of the Input/Output system. In the current model version it is assumed that all sectors has the same price index for investments - this follows from the assumption of identical deliveries. In a later version public investments will have a different price from private investments.

Imports in ADAM are not divided according to sectors, but according to 10 SITC-based import groups including foodstuff, raw oil, other energy, cars, other goods and services[25].

Imports of agriculture, construction, extraction, housing, sea transport and public services cannot be uniquely defined based on the 10 SITC-based groups. They are in the test version of the model set to 0. The national accounts have data for sectoral imports. It is no problem setting imports of construction, extraction, housing and public services to zero as these deliveries are very small. Imports of foodstuff is mainly from the foodstuff industry, but some come directly from the agriculture. We follow ADAM (where substitution is assumed to affect only the foodstuff industry) and place M01 as manufacturing (as foodstuff is a part of manufacturing in MAKRO). In regard to sea transport also follow ADAM and assume the substitution with import of both sea transport and other private services to be with other private services alone. Import of private services are defined as service import, M[ser]=Ms+Mt, energy imports can also be taken from ADAM, M[ene]=M3k+M3r+M3q, and the rest is defined as manufacturing, M[man]=M01+M2+M59+M7b+M7y.[26]

In MAKRO all supply and demand components both in values and quantities are in accordance to ADAMs data bank and the national accounts. The values of all deliveries

[25] In ADAM M01 substitute foodstuff industry - qf, M2 and M59 substitute manufacturing - qz, M3q substitutes oil refineries - ng and and Ms substitute private services - qz.

[26] There is no real substitution between import of cars M7b and domestic production as there is not produced cars in Denmark. M7b is a part of the broad import of manufactured goods and its delivery to the aggregated non-housing consumption good and service sector has a substitution effect to domestic production. This could be fixed by including a sector of goods that are imports only, which has 0 for all domestic inputs and consists of M7b and M7y - the latter being a small import group.

are also in accordance to ADAMs databank and the national accounts. The sectoral level from MAKRO is assumed to be the micro level. Each sector produces one product and the price of deliveries before taxes from this sector is the same for all demand components. This is not the case in the national accounts. So the IO cells in quantities are not the same in MAKRO and the national accounts. Imputed data based on this assumption is made in the iodata_ADAM.gms file. The aggregated quantities of sectoral imports and domestic production is scaled so the quantities of aggregated deliveries from all sectors and import components to specific demand components are the same in MAKRO and the national accounts. Except on the assumed micro level all aggregates are calculated as Laspeyres quantity and Paasche price indices.

All share parameters in the IO equations are statically calibrated so they are in accordance to the MAKROs IO data.

10.5 Sectoral investments and deliveries

The demand for investments is given by $I_{i,,s,t}$. It is modeled in the firms chapter.

We assume the deliveries to a unit of a given type of capital, k, is the same for all sectors. This implies that the decomposition of the investments in the IO-table is irrelevant to determined aggregated production and imports. Furthermore, it implies that the price of a given type of capital, k, is the same across sectors:

$$pI_{k,ds,t} = pI_{k,t}$$

This implies that the aggregated investments are given by a simple sum:

$$I_{k,t} = \sum_{ds} I_{k,ds,t}$$

It is this aggregated demand that is satisfied by deliveries from the different sectors. Without the above assumptions investments would be 4-dimensionally given by $I_{k,ds,s,t}$ now simple additivity gives it can be written as $I_{k,s,t}$ since $I_{k,ds,s,t} = \left(I_{k,ds,t}/\sum_{ds} I_{k,ds,t}\right) I_{k,s,t}$. In the national accounts the investment prices for the same capital goods differs across sector. This implies that the sectoral investment quantities in MAKRO does not match that from the national accounts. In this version we have, however, a correction factor on the sectoral prices $pI_{k,ds,t} = \lambda^{pI}_{k,t} pI_{k,t}$.

10.6 Balancing share parameters

Increasing for example $\mu^{R\_m}_{r,s,t}$ will increase material imports in sector r from sector s, $R\_m_{r,s,t}$. This means total imports will increase, but with exogenous $\mu^{R\_y}_{r,s,t}$ both inputs from domestic production, $R\_y_{r,s,t}$, and the aggregate of this and imports, $R\_s_{r,s,t}$, are unchanged. This gives some weird productivity effect in the aggregation. In order to avoid this we balance the share parameters endogenously:

$$\mu^{D\_s}_{d,s,t} = \lambda^{D}_{d,t} \frac{\mu^{D\_s0}_{d,s,t}}{\sum_s \mu^{D\_s0}_{d,s,t}}$$

$$\mu^{D\_y}_{d,s,t} = \lambda^{D\_s}_{d,s,t} \frac{\mu^{D\_y0}_{d,s,t}}{\mu^{D\_y0}_{d,s,t} + \mu^{D\_m0}_{d,s,t}}$$

$$\mu^{D\_m}_{d,s,t} = \lambda^{D\_s}_{d,s,t} \frac{\mu^{D\_m0}_{d,s,t}}{\mu^{D\_y0}_{d,s,t} + \mu^{D\_m0}_{d,s,t}}$$

for $D = R, C, G, I, X$, $d = r, c, g, i, x$. Increasing for example material imports, $R\_m_{r,s,t}$, via $\mu^{R\_m0}_{r,s,t}$ will simultaneously increase $R\_m_{r,s,t}$ and $\mu^{R\_m}_{r,s,t}$ and decrease $R\_y_{r,s,t}$ and $\mu^{R\_y}_{r,s,t}$. So it will for a given aggregated demand $R\_s_{r,s,t}$ simply shift the import share. In the calibration $\mu^{D\_s}_{d,s,t}$, $\mu^{D\_y}_{d,s,t}$ and $\mu^{D\_m}_{d,s,t}$ are determined as usual. It is imposed that $\sum_s \mu^{D\_s0}_{d,s,t} = 1$ and $\mu^{D\_y0}_{d,s,t} + \mu^{D\_m0}_{d,s,t} = 1$. Then $\mu^{D\_s0}_{d,s,t}$, $\mu^{D\_y0}_{d,s,t}$ and $\mu^{D\_m0}_{d,s,t}$ are determined proportional $\mu^{D\_s}_{d,s,t}$, $\mu^{D\_y}_{d,s,t}$ and $\mu^{D\_m}_{d,s,t}$ and $\lambda^{D}_{d,t} = \sum_s \mu^{D\_s}_{d,s,t}$ and $\lambda^{D\_s}_{d,s,t} = \mu^{D\_y}_{d,s,t} + \mu^{D\_m}_{d,s,t}$.

10.7 Public direct investments and public sales

Even though the share parameters, $\mu^{D\_s}_{d,s,t}$, $\mu^{D\_y}_{d,s,t}$ and $\mu^{D\_m}_{d,s,t}$ are endogenous they are as described above given by the exogenous variables $\mu^{D\_s0}_{d,s,t}$, $\mu^{D\_y0}_{d,s,t}$, $\mu^{D\_m0}_{d,s,t}$, $\lambda^{D}_{d,t}$ and $\lambda^{D\_s}_{d,s,t}$. There are, however, two exceptions where not even these parameters are exogenous. The exceptions are share parameter for deliveries from the public sector to private consumption, $\mu^{C\_y0}_{c,pub,t}$, and for deliveries from the public sector to investments, $\mu^{I\_y0}_{i,pub,t}$. These are endogenously given so that $C_{cSer,pub,t}$ and $I_{iM,pub,t}$ are given in accordance to:

$$p^{I}_{i,pub,t} I_{i,pub,t} = \mu^{I\_pub}_{i,t} PublicDirectInvestment^{v}_{t}$$

$$p^{C}_{c,pub,t} C_{c,pub,t} = \mu^{C\_pub}_{c,t} PublicSales^{v}_{t}$$

This formulation insures that the value of the sum of deliveries from the public sector to investments and private production are given by the two variables $PublicDirectInvestment^{v}_{t}$ and $PublicSales^{v}_{t}$. These two variables does not follow the general demand for investment and private consumption inputs. This implies that inputs from the public sector and hence public production will not be endogenously affected by private demand components.

11 Data and calibration

The parameters of the model can be divided into four categories: 1) calibrated parameters, 2) guesstimated parameters, 3) estimated parameters, and 4) focus parameters. All variables in the model that are data-covered must match the data or be explicitly stated not to and why.[27] For every variable matching the data one parameter is calibrated. In many equations more than one parameters enters an equation determining a variable. Here only one of the parameters are calibrated the other(s) are either guesstimated, estimated or impulse-response-matched. The ambition is to have parameters with estimated elasticities and properties consistent with the model. In the current version this is not the case for all parameters, but we should in time fulfill this ambition. The parameters used to match the yearly data should be the parameters that have the least influence on the marginal properties of the model. In a CES function elasticities will be estimated where as the share parameters will be calibrated. This is equivalent to calibrating the constant in a log-linear formulation or having an additive residual in an estimation on a log-linear form. Focus parameters are parameters critical to the properties of the model that are not easily estimated. These will be set in order to best possible match impulse-response functions from estimated SVARs.

Most variables only depend on temporary and lagged variables and parameters. These are statically calibrated independently of all values in the projection. In the projection most statically calibrated parameters are projected using ARIMA-processes, but some are projected constant.

Some variables depends on leaded variables. In order to match these variables it is necessary to use dynamic calibration where the whole projection is taken into account. In the dynamic calibration all the dynamically calibrated parameters have been given their values both historically and in the projection. Most of the exogenous variables are held constant in growth adjusted terms - i.e. most quantities grow with the Harrod-neutral growth rate, prices increase with the inflation rate, and values with both. The population projections (BFR) are included in the model - so demographics are not taken to be constant. Furthermore, the interest rate projection is included. In a future model version it is planned to use the projections for the export market growth and the foreign prices including the oil price. It is for most dynamically calibrated parameters assumed that they are constant for the whole projection.

11.1 Data

In the current version of the model we match the data from the data bank for the convergence program for 2018, KP18. The two major data sources are ADAM's databank and the forecast of demographic and labor market status (BFR). ADAM's databank is perfectly aligned with the Danish National Accounts. All data from ADAM's databank are read into a matrix called ADAM and all data from the BFR are read into a matrix called BFR. The government's most recent forecasts are read into a matrix called FM-BANK and register data from the Law Model is read into a matrix called GRUND. In this version of the model the only supplementing data are a few matrices from the Statbank from Statististics Denmark.

IO cells aka deliveries to demand components are imputed to be consistent with demand and supply from ADAM's databank in values and quantities and IO cells in values

[27]One example of an exception is the sectoral investments in quantities. These do not match data as MAKRO assume the same input structure for investments in a given capital good regardless of the receiving sector. Together with the assumption that all inputs from one sector has the same price this implies all sectors has the same investment price for the same type of capital. This is not consistent with the national accounts data and here we choose to keep the imputed sectoral investments instead of introducing an ad hoc adjustment factor as in ADAM.

and an assumption that all deliveries from a sector has the same before tax price to all demand components.

All price indices for demand and supply components (except deliveries to demand components) in the model are directly or indirectly matched to those from the national accounts - i.e. they are Paasche price indices. As most of them are also CES price indices the share parameters are calibrated to ensure consistency with both. In the forecast price indices are CES. In the appendix there is a short discussion of Paashce and CES price indices.

11.2 Focus parameters and elasticities

In this version of the model focus variables and elasticities are set according to values from other models or studies. The ambition is in time to estimate most of the elasticities in a set up fully consistent with the model. Some of the assumptions needs to be revisited before the model is officially used by the ministries.

Assumptions:

- The yearly labor augmented productivity growth rate should be set to 1.00 percent. This is in accordance with the steady state growth rate from the government forecast.
- The yearly foreign rate of inflation is set to 1.75 percent. This is in accordance with the steady state inflation rate from the government's forecast.
- The household portfolio should be set to 100 percent bank deposits for the young below 46 years and 20 percent bank deposits, 35 percent bonds, 4½ percent foreign stocks and 40½ percent domestic stocks for the older.
- The firm debt ratio is set to 60 percent except for housing where it is 0 percent and public services where it is 100 percent.
- The tax value depreciation rate is set to 15 percent for equipment and 4 percent for structures.
- The risk premium is set to 2 percent.
- Tax rates are set exogenously to be consistent with data from the Danish Ministry of Taxation

Elasticities based on estimations - will be changed with new estimation results:

- The elasticity of substitution between consumption groups is set to 0.3.
- Import elasticities are set to 1.25 for all sectors and all demand components.
- The elasticity of substitution between different sectoral inputs in the IO table is set to 0 as in ADAM.
- The export elasticities are set to 2.5 for all export groups.
- The elasticity of substitution between material inputs and other inputs are set to 0.67 for all sectors. This magnitude is in line with DREAM based on estimates made by Thomas Thomsen.
- The elasticity of substitution between capital and labor is set to 0.5. This is a bit higher than estimates from ADAM (and DREAM), but the current model has problems with calibrating historically with lower values. It is 0.25 in DREAM.

- The elasticity of substitution between capital of equipment and structures are set to 0.25. This is 0.6 in DREAM.

Focus variables - will be changed with SVAR matching:

- The relative risk aversion parameter is set to 2.
- The share of hand to mouth consumers is set to 0.511182.
- The habit formation parameter is set to 0.4.
- The consumers discount rate is set to 0.01.
- Installation costs are set to 0.6 and the neutral level is set to the actual level in the historical period and the depreciation rate plus the growth rate plus the inflation rate in the forecast period.
- Rigidity in the export equations is set to 0.5.
- The parameter from unemployment gap to wage growth is set to 0.346521.
- The rigidity parameter in the Calvo equation is set to 0.01.

11.3 Static calibration and the ARIMA processes - a typical example

How the different parameters are calibrated are described in the different chapters. Here we provide a more detailed example of static calibration. The ARIMA proces will be described in further detail in a later documentation.

Only a few equations in the model includes leaded variables. Most equations include only temporaneous and lagged variables. Share parameters, for example $\mu^{R\_m}_{r,s,t}$, in these equations can be calibrated on the basis of the historical data in a static calibration. Take the equation for domestically produced and imported materials to production, $R\_y_{r,s,t}$ and $R\_m_{r,s,t}$, as an example:

$$R\_y_{r,s,t} = \mu^{R\_y}_{r,s,t} R_{r,s,t} \left(\frac{P^{R\_y}_{r,s,t}}{P^{R}_{r,s,t}} \right)^{-\sigma^R_{r,s}}$$

$$R\_m_{r,s,t} = \mu^{R\_m}_{r,s,t} R_{r,s,t} \left(\frac{P^{R\_m}_{r,s,t}}{P^{R}_{r,s,t}} \right)^{-\sigma^R_{r,s}}$$

where $\mu^{R\_y}_{r,s,t}$ and $\mu^{R\_m}_{r,s,t}$ are endogenous share parameters,$R_{r,s,t}$ are the aggregates of inputs from both imports and domestic production,$\sigma^R_{r,s}$ are the elasticities of substitution between imports and domestic inputs, $P^{R\_y}_{r,s,t}$ and $P^{R\_m}_{r,s,t}$ are CES price indices for domestically produced and imported materials, and $P^R_{r,s,t}$ are the CES price indices for the aggregate of the two.

The share parameters, $\mu^{R\_y}_{r,s,t}$ and $\mu^{R\_m}_{r,s,t}$, are given as:

$$\mu^{R\_y}_{r,s,t} = \lambda^{R\_s}_{r,s,t} \frac{\mu^{R\_y0}_{r,s,t}}{\mu^{R\_y0}_{r,s,t} + \mu^{R\_m0}_{r,s,t}}$$

$$\mu^{R\_m}_{r,s,t} = \lambda^{R\_s}_{r,s,t} \frac{\mu^{R\_m0}_{r,s,t}}{\mu^{R\_y0}_{r,s,t} + \mu^{R\_m0}_{r,s,t}}$$

where $\lambda^{R\_s}_{r,s,t}$, $\mu^{R\_m0}_{r,s,t}$ and $\mu^{R\_y0}_{r,s,t}$ are exogenous parameters to be calibrated. The construction is to ensure that that when for example the import share, $\mu^{R\_m}_{r,s,t}$, increases, there will be a proportionate decrease in the share of domestically produced inputs, $\mu^{R\_m}_{r,s,t}$. This modeling is described under the Input/Output system. During calibration this leaves 5 calibrated parameters to be identified by only 4 equations in the model. We identity by adding a further equation that only appears in the calibration. It normalizes the sum of the two exogenous share parameters:

$$\mu^{R\_y0}_{r,s,t} + \mu^{R\_m0}_{r,s,t} = 1$$

The above 5 equations form their own closed block in the calibration - even though they are calibrated together with a lot of other equations and variables they are not influenced by these. This block is solved as 5 non-linear equations with 5 unknowns. The endogenous parameters are $\mu^{R\_y}_{r,s,t}$, $\mu^{R\_m}_{r,s,t}$, $\lambda^{R\_s}_{r,s,t}$, $\mu^{R\_m0}_{r,s,t}$ and $\mu^{R\_y0}_{r,s,t}$. Since only $\lambda^{R\_s}_{r,s,t}$, $\mu^{R\_m0}_{r,s,t}$ and $\mu^{R\_y0}_{r,s,t}$ are exogenous in the model - it is only the calibration of those that matters. Isolating the three calibrated parameters yield:

$$\lambda^{R\_s}_{r,s,t} = \mu^{R\_y}_{r,s,t} + \mu^{R\_m}_{r,s,t}$$

$$\mu^{R\_y0}_{r,s,t} = \frac{\mu^{R\_y}_{r,s,t}}{\lambda^{R\_s}_{r,s,t}} = \frac{\mu^{R\_y}_{r,s,t}}{\mu^{R\_y}_{r,s,t} + \mu^{R\_m}_{r,s,t}}$$

$$\mu^{R\_m0}_{r,s,t} = \frac{\mu^{R\_m}_{r,s,t}}{\lambda^{R\_s}_{r,s,t}} = \frac{\mu^{R\_m}_{r,s,t}}{\mu^{R\_y}_{r,s,t} + \mu^{R\_m}_{r,s,t}}$$

where the endogenous share variables are given by:

$$\mu^{R\_y}_{r,s,t} = \frac{R\_y_{r,s,t}}{R_{r,s,t}} \left(\frac{P^{R\_y}_{r,s,t}}{P^{R}_{r,s,t}} \right)^{\sigma^R_{r,s}}$$

$$\mu^{R\_m}_{r,s,t} = \frac{R\_m_{r,s,t}}{R_{r,s,t}} \left(\frac{P^{R\_m}_{r,s,t}}{P^{R}_{r,s,t}} \right)^{\sigma^R_{r,s}}$$

The static parameters are calibrated for as long a period as possible. In this case it is possible to calibrate from 1966. The calibrated static parameters are treated as data and projected using auto ARIMA-processes. The groups of parameters statically calibrated and projected by auto ARIMA-processes are defined in the file static_calibration.gms using groups defined in the different model files.

11.4 Dynamic calibration

Parameters that depend on leaded variables must be dynamically calibrated. All other parameters are set or determined by the static calibration and projected by ARIMA-processes. How the different parameters are calibrated are described in the different chapters. Here we give an overview of what variables are dynamically calibrated.

Age-dependent aggregated private consumption depends on leaded variables and are determined here. We match both aggregate consumption and an age-dependent consumption profile. This is done with an adjustment factor in the Euler equation for rational consumers. The adjustment factor is gradually removed during a ten year period.

The baseline growth rate in wages is calibrated to have the unemployment rate match data. After the calibration years the baseline growth rate of wages are set to 1 meaning that wages per effective unit grows with the rate of inflation when unemployment equals its structural rate.

The age-specific productivity are calibrated to have their profile match the register data and are scaled until the wage match the industrial wage from ADAM's databank normalized to 1 in 2010.

Given the Calvo parameter sector prices are fitted by calibrating the Calvo markups. As prices today depend on future prices - the calibrated Calvo markups today depend on future Calvo markups. The future markups are equal to the markup in the last period.

As usercost depends on future investments and lagged capital stock is used in the production, all parameters in the production function are dependent on future variables. This includes the share parameters for materials, the KL-aggregate, labor, the capital-aggregate and capital. It is assumed that the capital aggregate and the KL-aggregate in the calibrated periods are given by a Paasche chain index. This is also the case going forward as share parameters are held constant.

In the dynamic calibration the whole model is solved simultaneously for all periods ahead in order to calibrate the relevant parameters. This makes it more vulnerable to convergency problems. If the starting values of the parameters are too far from the correct values the model will not converge. In order to minimize this problem an old solution to the model is used as starting values. With new parameters or new names for old parameters it is possible to face convergence problems. In order to localize convergence problems and make better starting values for new variables a whole en suite of options are presented in the file dynamic_calibration. In a typical calibration they are, however, not needed.

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A Appendices

A.1 Government variables

Level	MAKRO-name	ADAM-name bill. kr. in 2015		Description (in danish)	Structural correction
.	vGovWealth	-wosk	-110	Nettoformue	
..	vGovAssets	woski	1000	Aktiver	
..	vGovDebt	wosku	1110	Passiver	
.	vrGov		12	Samlet afkast	
..	vGovNetInterest	tion2	-15	Nettorenteindtægter	7-year avg.
...	vGovIntOnAssets	tioii	16	Bruttorenteindtægter	
...	vGovIntOnDebt	ti_o_z	32	Bruttorenteudgifter	
..	vGovReval	-wosk+vosk(t-1)-tfn_o	27	Omvurderinger	
.	vGovBalance	tfn_o	-36	Faktisk saldo	
.	vGovPrimBalance	Tfn_o-tion2	-20	Primære saldo	
#	vGovRev	tf_z_o-tioii	1037	Samlede primære indtægter	
##	vtDirect	sy_o	617	Direkte skatter	
###	vtSource	syk(=ssy)	406	Kildeskatter	
####	vtBot			Bundskat	1.402
####	vtTop			Topskat	1.402
####	vtMun			Kommunal indkomstskat, sundhedsbidrag og amtskat	1.402
####	vtProp	ssyej	14	Ejendomsværdiskat	2.01
####	vtStock	ssya	21	Aktieskat	0.812
####	vtSourceRest			Øvrige kildeskatter	0.15
###	vtAM	sya	87	AM-bidrag	2.035
###	vtHHIncRest	syp	34	Andre personlige indkomstskatter	0.003
###	vtHHWeight	syv	8	Vægtafgift	1.44
###	vtCorp	syc	53	Selskabsskat	
####	vtCorpNorth	syc_e	2	Selskabsskat,nordsø	7-year avg.
####	vtCorpMain	syc_q	51	Selsskabsskat,øvrige	7-year avg.
###	vtPAL	sywp	23	PAL-skat	0
###	vtMedia	sym	4	Medie-licens	0.062
##	vtIndirect	spt_o	330	Indrekte-skatter	
###	vtVAT	spg	190	Moms	2.007
###	vtExciseDuty	spp+sppu	74	Punktafgifter	1.015
###	vtReg	spr	18	Registeringsafgifter	0
###	vtProduction	spz+spzu	48	Produktionsskatter	
####	vtLand	spzej	28	Ejendomsskatter(grundskyld)	0.687
####	vtFirmsWeight	spzv	3	Vægtafgift-fra-virksomheder	0
####	vtFirmsPayroll/vtFirmsWagetax	spzam	7	Lønsumsafgift	2.528
####	vtFirmsAM	spzaud	6	Arbejdsgiverbetalt AM-bidrag	1.172
####	vtProductionRest		3	Øvrige-produktionsskatter	0
###	vtCus	spm	3.3	Told	0
###	-vtEU	-spteu	-3.3	Indrekte-skatter-til-EU	0
##	vGovRevRest		85	Andre-offentlige-indtægter	
###	vtBequest	sk_h_o	5	Kapitalskatter(arveafgift)	7-year avg.
###	vGovDepr	invo1	59	Afskrivninger	0.776
###	vContribution	tp_h_o	20	Bidrag-til-sociale-ordninger	
####	vContUnemp	tpaf	11	Bidrag til a-kasse	7-year avg.
####	vContEarlyRet	tpef	3	Bidrag til efterløn	7-year avg.
####	vContFreeRest	tpr	1	Øvrige frivillige bidrag	7-year avg.
####	vContMandatory	stp_o	1	Obligatoriske bidrag	0.9
####	vContCivilServants	tpt_o	4	Tjenestemandspension	0.776
###	vGovReceiveForeign	tr_er_o+tk_e_o+tr_eu_o	2	Overførsler fra udlandet	7-year avg.
###	vGovReceiveHHFirms	trr_hc_o+tk_hc_o	-4	Overførsler fra DK	7-year avg.
###	vtChu	trks	6	Kirkeskat	1.325
###	vGovRent	Tirn_o	3	Jordrente	7-year avg.
###	vGovProfit	Tiuo_z_o	1	Overskud af offentlig virk.	7-year avg.

Level	MAKRO-name	ADAM-name bill. kr. in 2015		Description (in danish)	Structural correction
#	vGovExp	tf_o_z-ti_o_z	1058	Samlede primære udgifter	
##	vG	Co	521	Offentligt forbrug	0.776
##	vTrans	ty_o	352	Offentlige indkomstoverførsler	
###	vTransLabMarket		96	Arbejdsmarkedsrelaterede indkomstoverførsler	
####	vTransUnemp	Tyd+Tyrkk+Tyrki+Tyuad +tyuak/2	36		-15
####	vTransLabMarketRest	tyrku+tyrkr+tyuak/2 +Tyuly+tyms+tymb +tyusu+tymlf+Tyury+Tymr	60		0
###	vTransPensions		210	Pensioner o.lign.	
####	vTransOldAgePens	typfp+typfp_e	123	Folkepension	0.062
####	vTransWornPens	typfo+typfo_e	41	Førtidspension	0.062
####	vTransEarlyRet	typef	14	Overførsler til efterløn	0.062
####	vTransCivilServants	Typt	27	Tjenestemandspension	0.062
####	vTransPensOther	typfy+typr-typt	4	Andre pensionsoverførsler	0.062
###	vTransOther		46	Øvrige indkomstoverførsler	
####	vTransFamily	tyrbf	15	Børnefamilieydelse	0.026
####	vTransGreen	tyrgc	4	Overførsel til grøn check (fra 2013)	0
####	vTransHouseOther	tyrhs	4	Boligsikring	0.062
####	vTransHousePens	tyrhy	10	Boligydelse	0.062
####	vTransRestTaxable	Tyrrs	2	Øvrige overførsler, skattepligtige	0.062
####	vTransRestNontaxable	tyrrr	11	Øvrige overførsler, ikke skattepligtige	0.062
##	vGovInv	io1	73	Offentlige investeringer	7-year avg.
##	vGovSub	spu_o	41	Statens nettoudgifter til subsidier	-13.86
###	vSubProduct	sppu			
###	vSubProduction	spzu			
###	vSubEU	spueu			
##	vGovExpRest		65	Andre udgifter	
###	vGovLandRights	izn_o	-4	Køb af jord og rettigheder, netto	7-year avg.
###	vGovPaymentForeign	try_o_eu+trg_o_eu +trr_o_eu+tr_o_ef +tr_o_eg+trr_o_e+tk_o_e	44	Overførsler til udlandet	0
###	vGovPaymentFirms	tk_o_c	0	Kapitaloverførsler fra den offentlige sektor til selskaber	0
###	vGovPaymentHH	tk_o_h+tr_o_h	24	Overførsler fra den offentlige sektor til husholdningerne	0

A.2 Problem of the firm

The firm in sector s faces the following maximization problem:

$$\underset{R_{s,t},L_{s,t},I_{k,s,t}}{MAX}\left[\sum_{n=0}^{\infty}\frac{Div_{s,t+n}}{\prod_{m=0}^{n}\left(1+r_{t+m}^{firms}\right)}\right]$$

$$s.t.$$

$$\begin{array}{c} Div_{s,t}=\left(1-t_{s,t}^{Corp}\right)\left(\begin{array}{c} P_{s,t}^{Y}\left[\begin{array}{c} F\left(K_{k,s,t-1},L_{s,t},R_{s,t}\right)_{s} \\ -\sum_{k}\mu_{k,s,t}^{KInstCost}\frac{\left(I_{k,s,t}-\left(\delta_{k,s,t}+g_{t}\right)K_{k,s,t-1}\right)^{2}}{K_{k,s,t-1}}\end{array}\right] \\ -P_{s,t}^{L}L_{s,t}-P_{s,t}^{R}R_{s,t} \\ -\sum_{k}\delta_{k,s,t}^{Book}K_{s,t-1}^{book}-r_{t}^{firms}\mu_{s,t-1}^{FirmDebt}\sum_{k}P_{k,s,t-1}^{I}K_{k,s,t-1}\end{array}\right) \\ +\sum_{k}\delta_{k,s,t}^{Book}K_{k,s,t-1}^{book}-\sum_{k}P_{k,s,t}^{I}I_{k,s,t} \\ +\left(\mu_{s,t}^{FirmDebt}\sum_{k}P_{k,s,t}^{I}K_{k,s,t}-\mu_{s,t-1}^{FirmDebt}\sum_{k}P_{k,s,t-1}^{I}K_{k,s,t-1}\right) \\ K_{k,s,t}=\left(1-\delta_{k,s,t}\right)K_{k,s,t-1}+I_{k,s,t} \\ K_{k,s,t}^{book}=\left(1-\delta_{k,s,t-1}^{book}\right)K_{k,s,t-1}^{book}+I_{k,s,t}\end{array}$$

Note that the production tax $t_{s,t}^{Y}$ is not a part of the optimization part. This is not consistent, but at the moment it is not crucial and the production tax is going to be remodeled in a future model version. Letting $P_{k,s,t}^{sK}$ and $P_{k,s,t}^{I}P_{k,s,t}^{sKbook}$ denote the Lagrange multipliers we get the Lagrange function:

$$\mathcal{L}_{s,t}=\sum_{n=0}^{\infty}\frac{1}{\prod_{m=0}^{n}\left(1+r_{t+m}^{firms}\right)}$$

$$\left\{\begin{array}{c} \left(1-t_{s,t+n}^{Corp}\right)\left(\begin{array}{c} P_{s,t+n}^{Y}\left[\begin{array}{c} F\left(K_{k,s,t-1+n},L_{s,t+n},R_{s,t+n}\right)_{s} \\ -\sum_{k}\mu_{k,s,t+n}^{KInstCost}\frac{\left(I_{k,s,t+n}-\left(\delta_{k,s,t+n}+g_{t+n}\right)K_{k,s,t-1+n}\right)^{2}}{K_{k,s,t-1+n}}\end{array}\right] \\ -P_{s,t+n}^{L}L_{s,t+n}-P_{s,t+n}^{R}R_{s,t+n} \\ -\sum_{k}\delta_{k,s,t+n}^{Book}K_{s,t-1+n}^{book}-r_{t+n}^{firms}\mu_{s,t-1+n}^{FirmDebt}\sum_{k}P_{k,s,t-1+n}^{I}K_{k,s,t-1+n}\end{array}\right) \\ +\sum_{k}\delta_{k,s,t}^{Book}K_{k,s,t-1+n}^{book}-\sum_{k}P_{k,s,t+n}^{I}I_{k,s,t+n} \\ +\left(\mu_{s,t+n}^{FirmDebt}\sum_{k}P_{k,s,t+n}^{I}K_{k,s,t+n}-\mu_{s,t-1+n}^{FirmDebt}\sum_{k}P_{k,s,t-1+n}^{I}K_{k,s,t-1+n}\right) \\ +P_{k,s,t+n}^{sK}\quad\left(\left(1-\delta_{k,s,t+n}\right)K_{k,s,t-1+n}+I_{k,s,t+n}-K_{k,s,t+n}\right) \\ +P_{k,s,t+n}^{sKbook}\quad\left(\left(1-\delta_{k,s,t-1+n}^{book}\right)K_{k,s,t-1+n}^{book}+I_{k,s,t+n}-K_{k,s,t+n}^{book}\right)\end{array}\right\}$$

With regards to labor and material inputs the problem is static:

$$\frac{\partial F\left(K_{k,s,t-1},L_{s,t},R_{s,t}\right)_{s}}{\partial L_{s,t}}P_{s,t}^{Y}=P_{s,t}^{L}$$

$$\frac{\partial F\left(K_{k,s,t-1},L_{s,t},R_{s,t}\right)_{s}}{\partial R_{s,t}}P_{s,t}^{Y}=P_{s,t}^{R}$$

For the accumulation of real and book value capital, respectively, the first order condition with respect to investment is:

$$P_{k,s,t}^{sK}+P_{k,s,t}^{I}P_{k,s,t}^{sKbook}=P_{k,s,t}^{I}+\left(1-t_{s,t}^{Corp}\right)P_{k,s,t}^{I}\mu_{k,s,t}^{KInstCost}\frac{I_{k,s,t}-\left(\delta_{k,s,t}+g_{t}\right)K_{k,s,t-1}}{K_{k,s,t-1}}$$

this can be rewritten to our definition of $P_{k,s,t}^{sK}$ above.

The first order condition with respect to K_{t}^{Book} is:

$$P^{sKbook}_{k,s,t} = \frac{1}{1+r^{firms}_{t+1}} \left[\left(1-\delta^{book}_{k,s,t+1}\right) P^{sKbook}_{k,s,t+1} + t^{Corp}_{s,t+1} \delta^{book}_{k,s,t+1} \right]$$

while the first order condition with respect to K_t is:

$$\begin{aligned}
P^{sKbook}_{k,s,t} &= \tfrac{1}{1+r^{firms}_{t+1}} \left(1-t^{Corp}_{s,t}\right) P^{Y}_{s,t} \\
&\left[\begin{array}{c} \frac{\partial F(K_{k,s,t},L_{s,t+1},R_{s,t+1})_s}{\partial K_{k,s,t}} \\ +2\sum_k \left(\delta_{k,s,t+n}+g_{t+n}\right)^2 \mu^{KInstCost}_{k,s,t+n} \frac{I_{k,s,t+n}-(\delta_{k,s,t+n}+g_{t+n})K_{k,s,t-1+n}}{K_{k,s,t-1+n}} \\ +\sum_k \mu^{KInstCost}_{k,s,t+n} \frac{(I_{k,s,t+n}-(\delta_{k,s,t+n}+g_{t+n})K_{k,s,t-1+n})^2}{K^2_{k,s,t-1+n}} \end{array} \right] \\
&-\tfrac{1}{1+r^{firms}_{t+1}} \left(1-t^{Corp}_{s,t+1}\right) r^{firms}_{t+1} \mu^{FirmDebt}_{s,t} P^{I}_{k,s,t} \\
&+\tfrac{1}{1+r^{firms}_{t+1}} \mu^{FirmDebt}_{s,t} P^{I}_{k,s,t} \\
&+P^{sK}_{k,s,t+1} \quad (1-\delta_{k,s,t})
\end{aligned}$$

in optimum marginal cost equals marginal production, so user cost is derived as $P^{uK}_{k,s,t} = \frac{\partial F(K_{k,s,t},L_{s,t+1},R_{s,t+1})_s}{\partial K_{k,s,t}} P^{Y}_{s,t}$ given the first order condition.

The production function is a nested CES function:

$$Y^{gross}_{s,t} = \left(\left(\mu^{YKL}_{s,t}\right)^{1/\sigma^Y_s} KL_{s,t}^{\left(\sigma^Y_s-1\right)/\sigma^Y_s} + \left(\mu^{YR}_{s,t}\right)^{1/\sigma^Y_s} R_{s,t}^{\left(\sigma^Y_s-1\right)/\sigma^Y_s} \right)^{\sigma^Y_s/\left(\sigma^Y_s-1\right)}$$

$$KL_{s,t} = \left(\left(\mu^{KKL}_{s,t}\right)^{1/\sigma^{KL}_s} K_{s,t}^{\left(\sigma^{KL}_s-1\right)/\sigma^{KL}_s} + \left(\mu^{LKL}_{s,t}\right)^{1/\sigma^{KL}_s} \left(e^L_{s,t} L_{s,t}\right)^{\left(\sigma^{KL}_s-1\right)/\sigma^{KL}_s} \right)^{\sigma^{KL}_s/\left(\sigma^{KL}_s-1\right)}$$

$$K_{s,t} = \left(\left(\mu^{KK}_{iB,s,t}\right)^{1/\sigma^{KK}_s} K_{iB,s,t}^{\left(\sigma^{KK}_s-1\right)/\sigma^{KK}_s} + \left(\mu^{KK}_{iM,s,t}\right)^{1/\sigma^{KK}_s} K_{iM,s,t}^{\left(\sigma^{KK}_s-1\right)/\sigma^{KK}_s} \right)^{\sigma^{KK}_s/\left(\sigma^{KK}_s-1\right)}$$

The total costs of production is given by:

$$\begin{aligned}
P^{Y0}_{s,t} Y^{gross}_{s,t} &= P^R_{s,t} R_{s,t} + P^L_{s,t} L_{s,t} + \sum_k P^{uK}_{k,s,t-1} K_{k,s,t-1} \\
&= P^R_{s,t} R_{s,t} + P^L_{s,t} L_{s,t} + P^{uK}_{iB,s,t-1} K_{iB,s,t-1} + P^{uK}_{iM,s,t-1} K_{iM,s,t-1}
\end{aligned}$$

Given the user cost rate the cost optimization problem is three simple optimization problems:

$$\underset{R_{s,t},KL_{s,t}}{MIN} \left[P^R_{s,t} R_{s,t} + P^{KL}_{s,t} KL_{s,t} \right]$$
$$s.t.$$
$$Y^{gross}_{s,t} = \left(\left(\mu^{YKL}_{s,t}\right)^{1/\sigma^Y_s} KL_{s,t}^{\left(\sigma^Y_s-1\right)/\sigma^Y_s} + \left(\mu^{YR}_{s,t}\right)^{1/\sigma^Y_s} R_{s,t}^{\left(\sigma^Y_s-1\right)/\sigma^Y_s} \right)^{\sigma^Y_s/\left(\sigma^Y_s-1\right)}$$

$$\underset{K_{s,t},L_{s,t}}{MIN} \left[P^{uK}_{s,t} K_{s,t} + P^L_{s,t} L_{s,t} \right]$$
$$s.t.$$
$$KL_{s,t} = \left(\left(\mu^{KKL}_{s,t}\right)^{1/\sigma^{KL}_s} K_{s,t}^{\left(\sigma^{KL}_s-1\right)/\sigma^{KL}_s} + \left(\mu^{LKL}_{s,t}\right)^{1/\sigma^{KL}_s} \left(e^L_{s,t} L_{s,t}\right)^{\left(\sigma^{KL}_s-1\right)/\sigma^{KL}_s} \right)^{\sigma^{KL}_s/\left(\sigma^{KL}_s-1\right)}$$

$$\underset{K_{iB,s,t},K_{iM,s,t}}{MIN} \left[P^{uK}_{iB,s,t} K_{iB,s,t} + P^{uK}_{iM,s,t} K_{iM,s,t}\right]$$
$$s.t.$$
$$K_{s,t} = \left(\left(\mu^{KK}_{iB,s,t}\right)^{1/\sigma^{KK}_s} K_{iB,s,t}^{\left(\sigma^{KK}_s - 1\right)/\sigma^{KK}_s} + \left(\mu^{KK}_{iM,s,t}\right)^{1/\sigma^{KK}_s} K_{iM,s,t}^{\left(\sigma^{KK}_s - 1\right)/\sigma^{KK}_s}\right)^{\sigma^{KK}_s/\left(\sigma^{KK}_s - 1\right)}$$

The solution is given above and how to derive it is outlined in the appendix on solving CES nests.

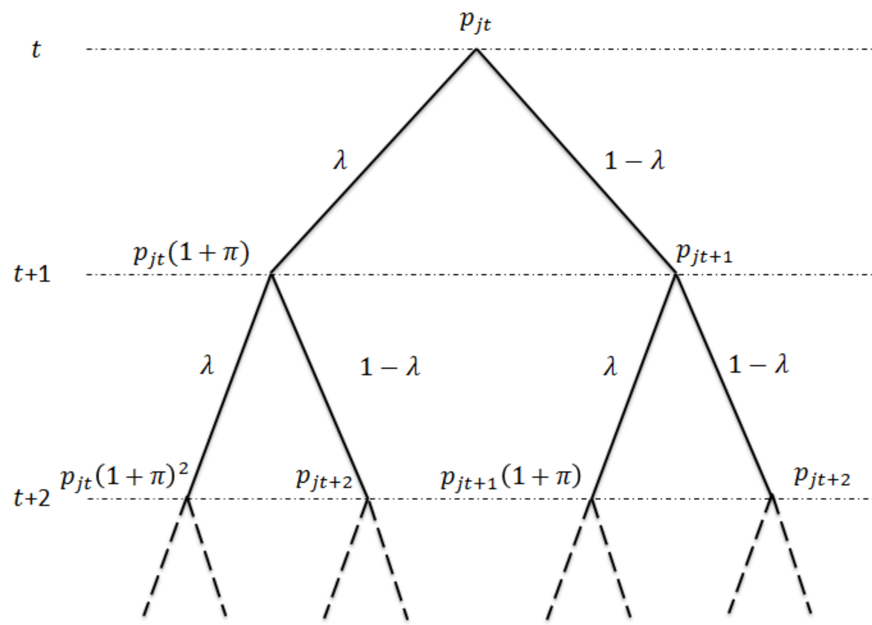
A.3 Deriving Calvo pricing

The point of departure is a lot of firms within each sector facing individually falling demand curves:

$$y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-E} Y_t$$

where $y_{jt}$is the demand for production by firm j at time t, p_{jt} is its price, P_t is the aggregate firm price (this is called $P^Y_{s,t}$ in the rest of the documentation. The suffix for sector and superfix for production is omitted in this appendix), Y_t (called $Y_{s,t}$ outside this appendix) is the aggregate production , and E is the elasticity of substitution (again sector specific, but suffix omitted).

In a given period a randomly chosen share $1-\lambda$ (called $1-\mu^{pYrigidity}_{s,t}$ outside this appendix) of the firms can change their prices. If unchanged the price follows the exogenous rate of inflation π (called gp_t in the code). Consider a firm resetting its price. It will make a full re-optimization. In absence of real rigidities the choice of price today will not influence the choice of price the next time the firm gets to reset its price. That is, when setting its price, the firm need only consider expected discounted profits in the particular case that it never gets to change price its again.A sketch of the situation facing a firm resetting price at time t is given below. From this one sees that current price p_{jt} only shows up on the far left branch.



Denote the expected discounted value of profits of the far left branch Π_j. For a firm resetting price at time t the value of this is given by:

$$E_t\left[\Pi_{jt-1}\right] = \sum_{s=t}^{\infty} \lambda^{s-t}\left(p_{jt}\left(1+\pi\right)^{s-t} y_{js} - p^O_s y_{js}\right)\frac{R_s}{R_{t-1}}$$

where p^O_t is marginally production cost (called $P^{Y0}_{s,t}$ outside this appendix), $R_t \equiv \prod_{s=1}^{t}\frac{1}{1+r_s}$ (r_t is called r^{firms}_t outside this appendix), and $y_{js} = \left(\frac{p_{jt}(1+\pi)^{s-t}}{P_s}\right)^{-E} Y_s$.

Before solving the problem it is rewritten. Define

$$\hat{p}_t \equiv \frac{p_t}{(1+\pi)^t}$$

and

$$\hat{R}_t \equiv R_t (1+\pi)^t \,,\; \hat{\Pi}_t \equiv \frac{\Pi_t}{(1+\pi)^t}$$

The problem is to choose $\hat{p}_{jt}$ to maximize

$$E_t\left[\hat{\Pi}_{jt-1}\right] = \sum_{s=t}^{\infty} \lambda^{s-t} \left(\hat{p}_{jt} y_{js} - \hat{p}^O_s x_{js}\right) \frac{\hat{R}_s}{\hat{R}_{t-1}}$$

subject to

$$y_{js} = \left(\frac{\hat{p}_{jt}}{\hat{P}_s}\right)^{-E} Y_s$$

This can be rewritten to

$$E_t\left[\hat{\Pi}_{jt-1}\right] = \sum_{s=t}^{\infty} \lambda^{s-t} \left(\hat{p}_{jt} - \hat{p}^O_s\right) \left(\frac{\hat{p}_{jt}}{\hat{P}_s}\right)^{-E} Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}}$$

such that

$$\begin{aligned}
\frac{\partial E_t\left[\hat{\Pi}_{jt-1}\right]}{\partial \hat{p}_{jt}} &= \sum_{s=t}^{\infty} \lambda^{s-t} \left[\left(\frac{\hat{p}_{jt}}{\hat{P}_s}\right)^{-E} - E\left(\hat{p}_{jt} - \hat{p}^O_s\right)\left(\frac{\hat{p}_{jt}}{\hat{P}_s}\right)^{-E-1} \frac{1}{\hat{P}_s}\right] Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}} \\
&= \sum_{s=t}^{\infty} \lambda^{s-t} \left[(1-E)\left(\frac{\hat{p}_{jt}}{\hat{P}_s}\right)^{-E} + E\left(\frac{\hat{p}_{jt}}{\hat{P}_s}\right)^{-E-1} \frac{\hat{p}^O_s}{\hat{P}_s}\right] Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}} \\
&= -\left(E-1\right)\hat{p}_{jt}^{-E} \sum_{s=t}^{\infty} \lambda^{s-t} \hat{P}_s^{E} Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}} + E \hat{p}_{jt}^{-E-1} \sum_{s=t}^{\infty} \lambda^{s-t} \hat{P}_s^{1+E} \frac{\hat{p}^O_s}{\hat{P}_s} Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}} \\
&= 0
\end{aligned}$$

Let $\hat{p}^*_t$ be the solution to this FOC (absence of real rigidities makes it equal for all firms). The optimal price is given by

$$\begin{aligned}
\hat{p}^*_t &= \frac{E}{E-1} \frac{\sum_{s=t}^{\infty} \lambda^{s-t} \hat{P}_s^{E} Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}} \hat{p}^O_s}{\sum_{s=t}^{\infty} \lambda^{s-t} \hat{P}_s^{E} Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}}} \\
&= \frac{E}{E-1} \frac{\Delta_t}{\Omega_t}
\end{aligned}$$

where

$$\Delta_t \equiv \sum_{s=t}^{\infty} \lambda^{s-t} \hat{P}_s^{E} Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}} \hat{p}^O_s$$

and

$$\Omega_t \equiv \sum_{s=t}^{\infty} \lambda^{s-t} \hat{P}_s^{E} Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}}$$

Rearranging yields

$$
\begin{aligned}
\Delta_t &= \sum_{s=t}^{\infty} \lambda^{s-t} \hat{P}_s^E Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}} \hat{p}_s^O \\
&= \hat{P}_t^E Y_t \frac{\hat{R}_t}{\hat{R}_{t-1}} \hat{p}_t^O + \sum_{s=t+1}^{\infty} \lambda^{s-t} \hat{P}_s^E Y_s \frac{\hat{R}_s}{\hat{R}_{t-1}} \hat{p}_s^O \\
&= \hat{P}_t^E Y_t \frac{\hat{R}_t}{\hat{R}_{t-1}} \hat{p}_t^O + \frac{\hat{R}_t}{\hat{R}_{t-1}} \lambda \sum_{s=t+1}^{\infty} \lambda^{s-(t+1)} \hat{P}_s^E Y_s \frac{\hat{R}_s}{\hat{R}_t} \hat{p}_s^O \\
&= \frac{1+\pi}{1+r_t} \left(\hat{P}_t^E Y_t \hat{p}_t^O + \lambda \Delta_{t+1} \right)
\end{aligned}
$$

such that

$$
\Delta_t = \frac{1+\pi}{1+r_t} \left(\hat{P}_t^E Y_t \hat{p}_t^O + \lambda \Delta_{t+1} \right)
$$

The same kinds of manipulations yield

$$
\Omega_t = \frac{1+\pi}{1+r_t} \left(\hat{P}_t^E Y_t + \lambda \Omega_{t+1} \right)
$$

Now for aggregation. Consider the aggregated CES price index P_t:

$$
P_t^{1-E} = \int_0^1 p_{jt}^{1-E} dj
$$

A share λ of the prices, p_{jt}, has just been changed and is equal to p_t^*. The rest is the 'old' prices. Following law of large number the aggregated price is

$$
P_t^{1-E} = \lambda \left(p_t^*\right)^{1-E} + (1-\lambda) P_{t-1}^{1-E}
$$

or

$$
P_t = \left[\lambda \left(p_t^*\right)^{1-E} + (1-\lambda) P_{t-1}^{1-E} \right]^{\frac{1}{1-E}}
$$

Inflation correcting:

$$
\frac{P_t}{(1+\pi)^t} = \left[\lambda \left(\frac{p_t^*}{(1+\pi)^t} \right)^{1-E} + (1-\lambda) \left(\frac{P_{t-1}}{(1+\pi)^t} \right)^{1-E} \right]^{\frac{1}{1-E}}
$$

such that

$$
\hat{P}_t = \left[\lambda \left(\hat{p}_t^*\right)^{1-E} + (1-\lambda) \left(\hat{P}_{t-1} \frac{1}{1+\pi} \right)^{1-E} \right]^{\frac{1}{1-E}}
$$

and

$$
\hat{P}_t^{1-E} = \lambda \left(\hat{p}_t^*\right)^{1-E} + (1-\lambda) \left(\hat{P}_{t-1} \frac{1}{1+\pi} \right)^{1-E} \tag{A.1}
$$

If the aggregated price index is a correct CES price index (which (A.1) is) is the aggregated output Y_t a CES aggregate of the production of all the firms:

$$
Y_t^{\frac{E-1}{E}} = \int_0^1 y_{jt}^{\frac{E-1}{E}} dj
$$

Y_t is determined by demand and $\hat{P}_t$ is determined by (A.1).

Inflation and growth corrected the Calvo equations can be written as:

$$\Delta_t = (1 + r_t) \left(P_t^E Y_t p_t^O + (1 + g) \lambda \Delta_{t+1} \right)$$

$$\Omega_t = (1 + r_t) \left(P_t^E Y_t + (1 + g) \lambda \Omega_{t+1} \right)$$

where r_t is the growth corrected real interest rate. Furthermore:

$$P_t^{1-E} = \lambda (p_t^*)^{1-E} + (1 - \lambda) \left(P_{t-1} \frac{1}{1 + \pi} \right)^{1-E}$$

where

$$p_t^* = \frac{E}{E-1} \frac{\Delta_t}{\Omega_t}$$

A.4 Solving CES nests

In solving a CES nest you have a set of simple optimization problems:

$$\underset{Q_1,Q_2}{MIN}\left[P_1Q_1 + P_2Q_2\right]$$
$$s.t.$$
$$Q = \left(\mu_1^{1/\sigma}Q_1^{(\sigma-1)/\sigma} + \mu_2^{1/\sigma}Q_2^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$$

The Lagrange function is:

$$\mathcal{L} = P_1Q_1 + P_2Q_2 - \lambda\left(\left(\mu_1^{1/\sigma}Q_1^{(\sigma-1)/\sigma} + \mu_2^{1/\sigma}Q_2^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)} - Q\right)$$

The first order conditions are:

$$\frac{\partial\mathcal{L}}{\partial Q_1} = P_1 - \lambda Q^{1/(\sigma-1)}\mu_1^{1/\sigma}Q_1^{-1/\sigma} \quad = 0$$

$$\frac{\partial\mathcal{L}}{\partial Q_2} = P_2 - \lambda Q^{1/(\sigma-1)}\mu_2^{1/\sigma}Q_2^{-1/\sigma} \quad = 0$$

$$\frac{\partial\mathcal{L}}{\partial \lambda} = Q - \left(\mu_1^{1/\sigma}Q_1^{(\sigma-1)/\sigma} + \mu_2^{1/\sigma}Q_2^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)} \quad = 0$$

The first two FOC yields:

$$\frac{Q_1}{Q_2} = \frac{\mu_1}{\mu_2}\left(\frac{P_1}{P_2}\right)^{-\sigma}$$

Isolating Q1 and inserting in the third FOC yields:

$$Q_2 = \mu_2 Q\left(\frac{P_2}{\left(\mu_1 P_1^{-(\sigma-1)} + \mu_2 P_2^{-(\sigma-1)}\right)^{1/(\sigma-1)}}\right)^{-\sigma}$$

Inserting this in the third FOC yields:

$$Q_1 = \mu_1 Q\left(\frac{P_1}{\left(\mu_1 P_1^{-(\sigma-1)} + \mu_2 P_2^{-(\sigma-1)}\right)^{1/(\sigma-1)}}\right)^{-\sigma}$$

The total cost is given by:

$$PQ = P_1Q_1 + P_2Q_2 = \left(\mu_1 P_1^{-(\sigma-1)} + \mu_2 P_2^{-(\sigma-1)}\right)^{-1/(\sigma-1)}Q$$

This yields:

$$P = \left(\mu_1 P_1^{-(\sigma-1)} + \mu_2 P_2^{-(\sigma-1)}\right)^{-1/(\sigma-1)}$$

Inserting yields:

$$Q_1 = \mu_1 Q\left(\frac{P_1}{P}\right)^{-\sigma}$$

$$Q_2 = \mu_2 Q\left(\frac{P_2}{P}\right)^{-\sigma}$$

A.5 On growth and inflation correction

Real growth and price inflation are introduced exogenously in the model. Variables in the model are understood as the residual of a common growth and inflation trend. Consider a price variable in the model. It is useful to decompose it into its stationary and inflation components. We typically write $p_t = (1+\pi_1)\,..\,(1+\pi_t)\times p_0$, but here we add an additional stationary component to have $p_t = (1+\pi_1)\,..\,(1+\pi_t)\times\zeta_t\times p_0$, which we then write as $p_t = (1+\pi_1)\,..\,(1+\pi_t)\times\hat{p}_t$. Then , when we have in the model an equation that contains prices at different dates in the model such as $p_t = bp_{t+1}$, we in fact work with

$$\hat{p}_t = b\left(1+\pi_{t+1}\right)\hat{p}_{t+1}$$

A standard example looks at how the capital accumulation is affected. From

$$K_t = (1-\delta)\,K_{t-1} + I_t$$

we write

$$\hat{K}_t = \frac{(1-\delta)}{1+g_t}\hat{K}_{t-1} + \hat{I}_t$$

and ignoring hats the steady state in these transformed variables implies $\frac{I}{K} = \frac{g+\delta}{1+g}$.

Consider now a more general example of how it is done.

The firm will maximize profits by choosing capital (where there is also book capital which is a regulation induced object)

$$\Pi_t = (1-\tau_t)\,p_tF_t - I_tp_t^I + \tau_t\delta^bK_{t-1}^b + q_t\left[\left(1-\delta^k\right)K_{t-1} + I_t - K_t\right] + \lambda_t\left[\left(1-\delta^b\right)K_{t-1}^b + I_tp_t^I - K_t^b\right]$$

With a nominal discount factor $\beta = 1/(1+i)$, the first order conditions with respect to investment, capital, and book-capital are

$$\begin{aligned} q_t &= p_t^I\left(1-\lambda_t\right) \\ \lambda_t &= \beta\lambda_{t+1}\left(1-\delta^b\right) + \beta\tau_{t+1}\delta^b \\ q_t &= \beta q_{t+1}\left(1-\delta^k\right) + \beta\left(1-\tau_{t+1}\right)p_{t+1}^k \end{aligned}$$

Now, lambda is a shadow price of a nominal quantity and so it does not grow (it has no corrections). However, q is the shadow price of a real quantity and so it must grow at the rate of inflation (because the investment price and the output price are growing at the rate of inflation).[28]

It is now useful to decompose our price variables into their stationary and inflation components so we are ready to rewrite everything with stationary variables as

$$\begin{aligned} \hat{q}_t &= \hat{p}_t^I\left(1-\lambda_t\right) \\ \lambda_t &= \beta\lambda_{t+1}\left(1-\delta^b\right) + \beta\tau_{t+1}\delta^b \\ \hat{q}_t &= \beta\hat{q}_{t+1}\left(1+\pi_{t+1}\right)\left(1-\delta^k\right) + \beta\left(1-\tau_{t+1}\right)\left(1+\pi_{t+1}\right)\hat{p}_{t+1}^k \end{aligned}$$

where all but the last inflation terms drop out of the system. In steady state (and abusing notation by dropping the hats) this yields $q = p^I(1-\lambda)$, and

$$\begin{aligned} \lambda(1+i) &= \lambda\left(1-\delta^b\right) + \tau\delta^b \\ q(1+i) &= q\left(1+\pi\right)\left(1-\delta^k\right) + (1-\tau)\left(1+\pi\right)p^k \end{aligned}$$

and rearranging we get $\lambda = \tau\delta^b/(i+\delta^b)$, and using the fact that $(1+i) = (1+r)(1+\pi)$, $q\left[r+\delta^k\right] = (1-\tau)\,p^k$, which yields

$$p^I\left(r+\delta^k - \frac{r+\delta^k}{i+\delta^b}\tau\delta^k\right) = (1-\tau)\,\hat{p}^k$$

[28] This may sound confusing. While prices grow at the exogenous rate of inflation, shadow prices of nominal quantities do not need an inflation correction.

A.6 CES and Paasche price indices

The CES demand for a quantity $Q_{i,t}$ is (for i=1,2) derived in appendix 1.2 and given by:

$$Q_{i,t} = \mu_{i,t} Q_t^{CES} \left(\frac{P_{i,t}}{P_t^{CES}} \right)^{-\sigma}$$

where $\mu_{i,t}$ is a share parameter, Q_t^{CES} is the CES aggregate of the demand components, $P_{i,t}$ is the the price for demand component i, P_t^{CES} is the CES price index and σ is the elasticity of substitution. The budget constraint gives:

$$P_t^{CES} Q_t^{CES} = \sum_i P_{i,t} Q_{i,t}$$

The above two equations implicitly gives the CES price index. In MAKRO all CES price indices are implicitly given. Inserting and writing the CES price index explicitly gives:

$$P_t^{CES} = \left(\sum_i \mu_{i,t} P_{i,t}^{-(\sigma-1)} \right)^{-1/(\sigma-1)}$$

The national accounts use Laspeyres chain index for quantities:

$$P_{t-1}^{PCI} Q_t^{LCI} = \sum_i P_{i,t-1} Q_{i,t}$$

where P_t^{PCI} is a Paasche chain index for prices and Q_t^{LCI} is a Laspeyres chain index for quantities. Together with the budget constraint this yields a Paasche chain index for prices:

$$P_t^{PCI} = P_{t-1}^{PCI} \frac{\sum_i P_{i,t} Q_{i,t}}{\sum_i P_{i,t-1} Q_{i,t}}$$

For historical data the $\mu_{i,t}$ are calibrated so either $P_t^{CES} = P_t^{PCI}$ or $Q_t^{CES} = Q_t^{LCI}$ together with the budget constraint the one implies the other and we need not to distinguish. This implies:

$$\mu_{i,t0} = \frac{Q_{i,t0}}{Q_{t0}^{LCI}} \left(\frac{P_{i,t0}}{P_{t0}^{PCI}} \right)^{\sigma}$$

where t0 is the last year of the calibration period.

In the forecast period our CES prices will only equal Paasche chain prices if:

$$P_{t-1}^{CES} Q_t^{CES} = \sum_i P_{i,t-1} Q_{i,t}$$

$$P_{t-1}^{CES} Q_t^{CES} = \sum_i P_{i,t-1} \mu_{i,t} Q_t^{CES} \left(\frac{P_{i,t}}{P_t^{CES}} \right)^{-\sigma}$$

$$P_{t-1}^{CES} = \left(P_t^{CES} \right)^{\sigma} \sum_i \mu_{i,t} P_{i,t-1} P_{i,t}^{-\sigma}$$

$$\left(\sum_i \mu_{i,t-1} P_{i,t-1}^{-(\sigma-1)} \right)^{-1/(\sigma-1)} = \left(\sum_i \mu_{i,t} P_{i,t}^{-(\sigma-1)} \right)^{-\sigma/(\sigma-1)} \sum_i \mu_{i,t} P_{i,t-1} P_{i,t}^{-\sigma}$$

This will generally not be the case. The Paasche price index is a superlative price index - so the deviation from a chain index will not increase over time, but we should never expect to hit exactly.

We have put restrictions on the share parameters not to catch productivity gains in the share parameters. Instead of balancing with fixed coefficient sums you can balance by enforcing the CES price aggregates to equal the Paasche price aggregates. This is done by endogenizing λ_t in the equation:

$$\mu_{i,t} = \lambda_t \frac{\mu_{i,t}^0}{\sum_i \mu_{i,t}^0}$$

and adding an equation stating:

$$P_{t-1}^{CES} Q_t^{CES} = \sum_i P_{i,t-1} Q_{i,t}$$