General Equilibrium Tax Policy with Hyperbolic Consumers *

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Abstract

Recently David Laibson and others have argued in favor of using hyperbolic discount functions. The purpose of this paper is to investigate whether conventional wisdom, based on the standard model with exponential discounting, also holds in the case where consumers have hyperbolic discount functions. In other words do hyperbolic preferences matter for practical policy evaluation?

Within the framework of a suitably modified standard General Equilibrium model à la Auerbach and Kotlikoff, this is done by simulations of both fundamental changes in the tax base, as well as more marginal experiments comparing the excess burden of taxation. Based on the simulations it turns out that the answer to the question is a *maybe*: if preferences are sufficiently hyperbolic then policy conclusions change. Unfortunately this degree of hyperbolicness in the discounting function is at the level that is considered realistic by empirical studies.

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1 Introduction

Recent papers by Laibson (1996, 1997, 1998, 2001) and others have argued in favor of using a hyperbolic discount function for consumers - in contrast to the traditional exponential discount function. The hyperbolic discounting function allows for the suggestion by Strotz (1956) that discount rates are higher in the short-run than in the long-run[1], and the idea suggested by Akerlof (1991) that agents make future plans that they subsequently want to alter. The hyperbolic speci…cation has two advantages: …rst of all, some empirical evidence (Ainslie, 1992) supports the idea that consumers actually behave in this time-inconsistent manner, and secondly, according to (Laibson, 1998) the model seems to be able to explain some anormalies that cannot be explained by the standard life-cycle model with consumers with exponential discount factors.

This paper does not present evidence in favor of either of the two models, but tries to compare them by means of policy experiments. This means comparing simulations of fundamental changes in the tax base, as well as more marginal experiments comparing the excess burden of taxation. In a sense this is the ultimate test of importance: if similar policy experiments show almost identical results in a model with and without hyperbolic discounting, this indicates that the general framework is relatively robust. If on the other hand results are markedly di¤erent in the two situations, then further investigations are needed to determine whether the consumers' discount factors are in fact hyperbolic - and in a¢rmative case this should change the shape of applied macroeconomic modeling.

Why would we expect that di¤erent results to be obtained in the two cases? Clearly consumer behavior di¤ers in the two models. First of all the consumer's savings behavior is signi…cantly di¤erent: consumers whose discount factors are exponential have higher savings over the life cycle, whereas the hyperbolic consumers are less patient and get tempted to consume more and save less. This fundamentally di¤erent savings behavior turns out to be important for understanding the di¤erences. Secondly labor supply over the life cycle di¤ers: the exponentially discounting consumers retire early

[1] The example from Thaler (1981) explains the idea clearly: "When two rewards are both far away in time, decision-makers act relatively patiently (e.g., I prefer two apples in 101 days rather than 1 apple in 100 days). But when both rewards are brought forward in time, preferences exhibit a reversal, re‡ecting much more impatience (I prefer one apple now, rather than two tomorrow)" (quoted from Angeletos, Laibson, Repetto, Tobacman and Weinberg (2000, p. 5)).

and use savings to …nance their consumption during the retirement phase, whereas the hyperbolic consumers have less saved for retirement, and end up working more (i.e. work more hours and retire later). Therefore taxation of labor income have di¤erent consequences for the two types of consumers, since they have di¤erent labor supply behavior.

This paper compares the two types of preferences using a suitably modi…ed standard General Equilibrium model à la Auerbach and Kotliko¤ (1987). Overlapping generations of consumers face a consumption/savings and an endogenous labor supply decision in each period of their lives. The consumers live for 55 periods and do not face uncertainty. The government sector levies taxes on income and use the revenue for public expenditures. The production side is standard: …rms produce using capital and labor according to a constant return to scale technology.

The contribution of this paper is …rst of all that the analysis is carried out in general equilibrium, as opposed to previous partial equilibrium work. Secondly, the model introduces an endogenous labor supply decision - an addition which will in‡uence the general equilibrium simulations. Thirdly, the perspective of the paper is new: previous work has not focused on whether the conventional wisdom from the standard model also holds true if consumers are hyperbolic, but this important topic is in focus here.

The simulations show that the answer to the question do hyperbolic preferences matter? is not clear - it depends on the "degree of hyperbolicness". For speci…cations close to the standard model, the results do not change too much. But if the "degree of hyperbolicness" is large, then the recommendations based on the model may change - and conventional wisdom does not hold in all cases.

2 Model

This section describes the model used. On purpose the model is kept simple, and only the consumer side di¤ers from the usual set-up (Auerbach and Kotliko¤, 1987).

2.1 Consumers

The consumer's time-inconsistent preferences used are modelled along the lines of Phelps and Pollak (1968). Consumers live 55 periods, and their preferences change each period in a manner such that an individual with the age i has the utility function

$$U_i = \frac{1}{1 - \frac{1}{\gamma}} \left[u_i^{(1 - 1/\gamma)} + \beta \sum_{t=i+1}^{55} (1+\delta)^{-(t-1)} u_t^{(1-1/\gamma)} \right] \tag{1}$$

where utility from the current period is not discounted, and all future utility is discounted both by the usual geometric series given by $(1+\delta)^{-(t-1)}$ and the constant β: These discounted utilities are aggregated using a CES-type index where the intertemporal elasticity of substitution for the household is given by γ.

In each period the utility is given by the annual utility function u_t, which is a function over consumption and leisure and defined by the CES index

$$u_t = \left[c_t^{(1-1/\rho)} + \alpha (1 - l_t)^{(1-1/\rho)} \right]^{1/(1-1/\rho)} \tag{2}$$

where c_t is consumption in period t, l_t is labor supply in period t, and where α represents the household's preferences for leisure relative to consumption, and where ρ is the elasticity of substitution between leisure and consumption.

The degree of time-inconsistency in (1) depends on the size of β. When $\beta = 1$ the model reduces to the standard time-consistent case where discounting occurs geometrically. Since optimal solution to the consumer's problem is consistent the problem needs only to be solved once: at the first period. When $\beta < 1$ the consumer's choices at different ages are not consistent: when the consumer solves his problem in the first periods he makes future plans that he subsequently will want to alter[2]. For this reason the consumer's problem needs to be solved not only at the first period, but resolved each year.

[2] In the sense of Angeletos et al. (2000) the agents are naive, and make current decisions based on the optimistic belief that they later will follow the current decision. Notice that these agents are fully optimizing and rational at each moment in time - their plans however are not consisient over time.

In contrast to this stands the sophisticated agent that foresees that he later will want to deviate from the current plan. An agent of the latter type will therefore seek to limit his later actions by placing savings in illiquid assets, irreversible retirement savings plans etc. The present model contains no commitment device that allows this.

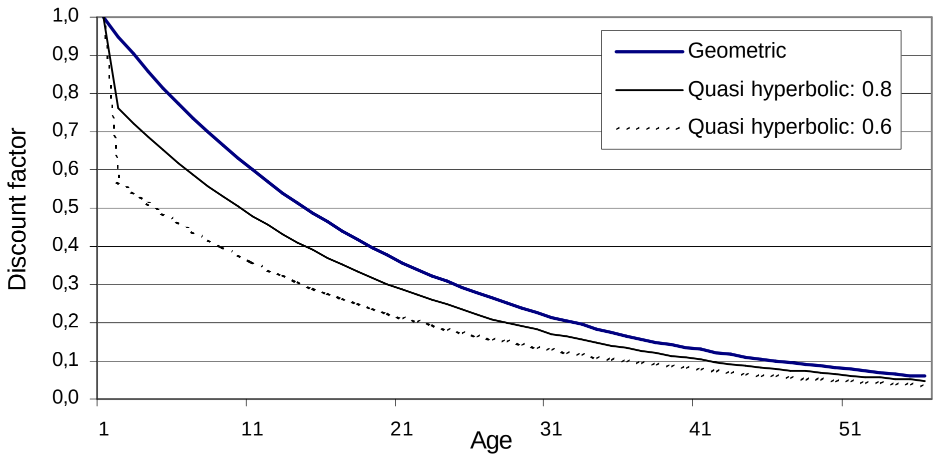

Figure 1. Overall discount factors for various values of β.

The impact of β on the discount term $\beta(1+\delta)^{-(t-1)}$ is shown below. Figure 1 shows what the discount factor looks like for an individual aged 1 (at time 1), in the situation where $\beta = 1$; $\beta = 0.8$ and $\beta = 0.6$ (in all cases $\delta = 0.05$). When $\beta < 1$ the figure illustrates the time-inconsistent property of the hyperbolic preferences[3]. In period 1 the consumer weighs utility in period 1 and 2 with the discount rate $\beta\frac{1}{(1+\delta)}$; but compares utility in period 2 and 3 with the factor $\frac{1}{(1+\delta)}$. However, after the first period he weighs consumption in period 2 and 3 with $\beta\frac{1}{(1+\delta)}$ instead of $\frac{1}{(1+\delta)}$.

2.2 The rest of the economy

The rest of the closed economy is standard and identical to Auerbach and Kotlikoff (1987). There is a single good, that is produced using capital and labor subject to a constant-returns-to-scale technology. Production takes place using the CES production function:

$$Y(K;L) = A\left[\varepsilon K^{(1-1/\sigma)} + (1-\varepsilon)L^{(1-1/\sigma)}\right]^{1/(1-1/\sigma)} \qquad (3)$$

where K and L are capital and labor in the period, Y is output, A is a scaling constant, ε is a capital-intensity parameter and σ is the elasticity of

[3] Strictly speaking the preferences above are what Laibson (1998) label quasi-hyperbolic, which is a discrete-time version of a hyperbolic discount function. We will, however, ignore this difference, and use the term hyperbolic throughout this paper.

substitution between K and L. Since we assume no adjustment costs in K or L, nor any depreciation of capital, we have the standard result that the gross wages must equal the marginal revenue product of labor (measured in efficiency units):

$$w = (1-\epsilon) A \left[\epsilon K^{(1-1/\sigma)} + (1-\epsilon) L^{(1-1/\sigma)} \right]^{1/(1-1/\sigma)} L^{-1/\sigma} \quad (4)$$

and the interest rate equals the marginal revenue product of capital:

$$r = \epsilon A \left[\epsilon K^{(1-1/\sigma)} + (1-\epsilon) L^{(1-1/\sigma)} \right]^{1/(1-1/\sigma)} K^{-1/\sigma} \quad (5)$$

The government sector is kept very simple. Government revenue is raised by taxation of labor income, interest income and a consumption tax. For each generation the tax payments to the government is

$$TAX_j = (1+\tau_a) r a_{j-1} + (1+\tau_c) c_j + (1+\tau_l) w e_j l_j$$

where a_{j-1} is the period's asset holdings at the beginning of the period (assets that pay interest in the period in question), and e_t is the productivity factor for t-year old individuals[4]. The revenue from taxation is not transferred back to the consumers, but is consumed. This is in line with the methodology used by Auerbach and Kotlikoff (1987), but differs from what is often assumed in Computable General Equilibrium models.

3 Calibration and simulation

The model outlined above is kept as close to Auerbach and Kotlikoff (1987) as possible, and the same principle will be applied in the calibration. Obvious parameters that relate to the hyperbolic discounting need to be taken from an other source, and the choice here is the paper by Laibson, Repetto and Tobacman (2000).

Laibson et al. (2000, page 28) note that "most of the experimental evidence suggests that the one-year discount factor is at least 30%-40%"[5], and use

[4] This is a hump-shaped profile over the life-cycle that gives an earnings profile that peaks after 30 years at the labor market (corresponding to real age 50) at wages that are 45% higher than at age 1 (when entering the labor market). This is the same profile as used by Auerbach and Kotlikoff (1987), which in turn originate from a study by Welch (1979).

[5] The authors base this on the review study by Ainslie (1992).

$\beta = 0.7$ in their simulations. In this paper simulations will be performed for the following values of β: 0.99; 0.95; 0.90; 0.80; 0.70 and 0.60. This covers the range mentioned by Laibson et al. (2000), as well as values of β that make consumer behavior closer to the standard model (in which $\beta = 1.0$).

3.1 Simulation

Since it is not possible to write down a closed form solution to the problem outlined above, the model is simulated numerically; the equilibrium is found iteratively using a Gauss-Seidel algorithm - similar to Auerbach and Kotlikoff (1987). Finding the equilibrium solution can be separated in two problems: the first problem is to solve the consumer's time-inconsistent problem, and the second problem is to find the general equilibrium for the economy.

3.1.1 Solving the consumer's problem

To solve the consumer's problem we need a set of exogenous factor prices, w and r. The consumer's problem is to maximize (1), subject to the present value budget constraint:

$$\sum_{t=1}^{55} (1+r)^{-(t-1)} \left[w e_t l_t - p c_t \right] = 0 \tag{6}$$

as well as the constraints

$$c_t \geq 0 \ (\forall t)$$
$$1 \geq l_t \geq 0 \ (\forall t)$$

This constrained maximization problem[6] is solved using GAMS. Since the solution is time-inconsistent, it is necessary to maximize U_i (equation (1)) for $i \in \{1, \ldots, 55\}$ - i.e. solve for the remaining life-time for all agents.

[6] Alternatively, the solution to the consumer's problem could be found solving a system of first order conditions. However, since this system of equations would generate the same solution, and have to be solved numerically anyway, the direct maximization approach is chosen because it is easier to implement.

3.1.2 Equilibrium iterations

In finding the equilibrium solution for the economy, we perform the following steps until convergence:

1. Make a guess for the aggregate values of K and L in the economy.
2. Given these values for K and L, use the firm's first order conditions (equations (4) and (5)) to determine the associated factor prices, r and w.
3. Solve the consumers problem. Since consumers are time inconsistent this means maximizing U_i (equation (1)) for $i \in \{1, \ldots, 55\}$ subject to the budget constraint (equation (6)).
4. Determine the new aggregate factor inputs $\hat{K}$ and $\hat{L}$. This is simply done by making a summation over the individual labor supply and savings for each generation (since labor across ages differ in efficiency due to the age-dependent productivity term e_j - see equation (6), the total labor supply is calculated as the sum of the individual labor supplies for each age-group).
5. If $K = \hat{K}$ and $L = \hat{L}$ the process has converged and we are done: these equilibrium values give factor prices that are consistent solutions to the problems of the producer and consumer, and market clearing.

   Otherwise calculate updated guesses, K^{new} and L^{new}, as a convex combination of the old (initial) values and the new aggregate values. Here this is done by using $K^{new} = \frac{1}{2}K + \frac{1}{2}\hat{K}$ and $L^{new} = \frac{1}{2}L + \frac{1}{2}\hat{L}$. Go to step 2 and use these values in the firms' first order conditions.[7]

3.1.3 Numerical implementation

The model described above is implemented in GAMS. The consumers problem is solved using Conopt2 (Drud, 1985), with the Gauss-Seidel updating procedure outlined above in a loop.

[7] In simulations where government revenue is constant, and some tax rates are endogenous, the tax rates are updated in a similar fashion.

4 Structural tax reform

Having presented the hyperbolic and the standard models we can now perform the first set of similar policy experiments on the two models: implement a fundamental tax reform. As mentioned in the introduction this is where the exercise gets interesting: will the "usual results" hold in the hyperbolic economy? And if there are differences between the standard and the hyperbolic economy, how will the results depend on the size of β in the utility function (equation (1))?

A comparison of the properties of different tax bases is a classical first step when comparing different models. This section presents an analysis of various tax bases, both in the standard model and the hyperbolic model. The analysis and the tax bases compared will be the same as in Auerbach and Kotlikoff (1987), namely comparing the income tax system to three other regimes where labor, interest and consumption are the tax bases.

4.1 Results

Table 1 below compares various key information for the different economies under consideration; economies that differ only in their βs (as far as exogenous variables are concerned). The column with the label "Std." represents the standard model (β=1). The table shows the size of production, consumption, labor supply and capital stock, the savings rate in percent, the utility for a newborn agent as well as the factor price ratio (w=r) in the benchmark stationary state equilibrium with income taxation:

	Std.	β=0.99	β=0.95	β=0.90	β=0.80	β=0.70	β=0.60
Production	25.53	25.44	25.09	24.63	23.64	22.47	21.07
Consumption	20.70	20.64	20.38	20.06	19.35	18.50	17.44
Gov. cons.	3.82	3.80	3.75	3.69	3.54	3.36	3.15
Savings rate	3.96	3.92	3.77	3.58	3.18	2.74	2.27
Labor supply	19.12	19.12	19.11	19.11	19.15	19.22	19.31
Capital stock	95.75	94.48	89.41	82.99	69.95	56.62	43.06
Fact. prices	15.01	14.82	14.03	13.03	10.96	8.84	6.69

Table 1. Benchmark equilibria under income taxation (absolute values).

The table below shows the same numbers in index values:

	Std.	¯=0.99	¯=0.95	¯=0.90	¯=0.80	¯=0.70	¯=0.60
Production	100.0	99.65	98.25	96.45	92.56	88.00	82.49
Consumption	100.0	99.70	98.47	96.89	93.45	89.33	84.21
Gov. cons.	100.0	99.65	98.25	96.45	92.56	88.00	82.49
Savings rate	100.0	98.99	95.20	90.40	80.30	69.19	57.32
Labor supply	100.0	99.98	99.93	99.94	100.15	100.47	100.97
Capital stock	100.0	98.68	93.38	86.68	73.06	59.14	44.98
Fact. prices	100.0	98.73	93.47	86.81	73.02	58.90	44.57

Table 2. Benchmark equilibria under income taxation (index values: standard model=100).

The characteristics of the economies under comparison are markedly different. Output as well as consumption goes down when ¯ decreases. The factor supply shows an interesting tendency: labor supply is relatively constant (and is even higher than the standard case for very low values of ¯), whereas the capital stock decrease drastically when ¯ decreases - an e¤ect that is caused by the decreasing "bene…ts" from savings by the hyperbolic consumers. This decrease in savings take place despite the large increase in the interest rate: notice the very di¤erent equilibrium factor price ratios ($\frac{w}{r}$) for di¤ering ¯s.

4.1.1 Labor income taxation

The …rst fundamental tax reform under consideration is a move to labor income taxation - which relative to the base case means abandoning taxation of capital income. Table 3 below shows the results of the reform as an index relative to the base case (income taxation) scenario, but with the size of the labor income tax in levels:

	Std.	¯=0.99	¯=0.95	¯=0.90	¯=0.80	¯=0.70	¯=0.60
Production	99.37	99.37	99.38	99.34	99.38	99.54	99.69
Consumption	98.94	98.95	98.96	98.92	98.97	99.20	99.41
Lab.inc.tax	20.08	20.08	20.07	20.08	20.08	20.04	20.02
Labor supply	97.23	97.21	97.14	96.99	96.82	96.78	96.74
Capital stock	106.10	106.17	106.42	106.73	107.47	108.33	109.09
Utility	97.34	97.39	97.56	97.79	98.30	98.91	99.99

Table 3. Labor income tax reform.

With the labor income tax as the only source of revenue, the constant marginal rate must be increased from 15% to 20.08% for total revenue to remain constant (the row "Lab.inc.tax"). This reform does not surprisingly strengthen capital accumulation, and lower labor supply.

From an overall welfare perspective the e¤ect is negative: utility for the representative agent decreases in the benchmark with 2.66% (index 97.34). In the hyperbolic economies (¯ < 1) there is a smaller decrease in utility: with ¯ = 0:9 the decrease in utility from a move from income taxation to labor income taxation is 2.21% (index 97.79) - which is a 17% improvement compared to the standard case (17%=(2.66-2.21)/2.66). For higher ¯s this e¤ect is more pronounced: in the case of ¯=0.7, the value used by Laibson et al. (2000), the decrease in utility is only 1.09%. The e¤ects on the capital stock also depends on ¯: in the standard case the change to labor income taxation increases capital stock by 6.1%, whereas we with ¯=0.9 and ¯=0.7 experience increases on 6.73% and 8.33%. Finally notice a slightly more negative e¤ect on labor supply for higher ¯s: in the standard case labor supply decreases 2.77%, but for ¯=0.9 and ¯=0.7 the decrease is 3.10% and 3.22%.

4.1.2 Capital income taxation

The second fundamental policy experiment in this paper is a change from income to capital income taxation - in other words removing the labor income tax. Table 4 below shows the results of the reform as an index relative to the base case scenario (income taxation):

	Std.	¯=0.99	¯=0.95	¯=0.90	¯=0.80	¯=0.70	¯=0.60
Production	95.76	95.69	95.40	94.98	93.98	92.96	92.00
Cons.	96.43	96.35	95.98	95.45	94.16	92.83	91.52
Cap.inc.tax	63.13	63.17	63.36	63.64	64.32	65.03	65.71
Lab. supply	109.50	109.56	109.79	110.02	110.38	110.77	111.17
Cap. stock	64.05	63.76	62.59	61.13	57.99	54.95	52.13
Utility	96.56	96.31	95.26	93.86	90.66	87.04	83.07

Table 4. Capital income tax reform.

Notice the rather large increase in the capital income tax necessary to generate the same revenue as the removed labor income tax: the tax rate on capital income increases from 15% to 63.13% in the standard case (the row called "Cap.inc.tax"), and with an even higher increase in the cases where ¯ < 1. This signi…cantly higher tax on capital income a¤ects the size of the capital stock in the economy: in the new stationary state, the capital stock has dropped 35.95%. For the hyperbolic economies, where the capital stock is already lower because the utility function biases the consumer to save less, this drop is even higher; with β=0.9 and β=0.7 we experience decreases of 38.87% and 45.05%. The removal of labor income taxation makes the supply of the other factor go up: in the standard case labor supply increases 9.50%, and in the hyperbolic economies this increase is slightly higher: with β=0.9 and β=0.7 the increase is respectively 10.02% and 10.7%.

This twist in the factor supply affects production and welfare. With the lower input of capital, and the higher input of labor, the overall output goes down with 4.24% in the standard case - a decrease that for lower values of β is even more pronounced, and in the case of β=0.7 is 7.05%. However since labor supply went up, and consumers derive disutility from working, we would expect overall utility to decrease even more than consumption, and this turns out to be a correct conjecture. In the standard case utility goes down 3.44%, but this effect is stronger when consumers are hyperbolic: with β=0.9 and β=0.7 the decrease is respectively 6.14% and 12.96%.

4.1.3 Consumption income taxation

The final fundamental reform considered is a change to consumption taxation. This means removing taxation on income altogether, and replace the revenue with a consumption tax. Table 5 below shows the results of the reform as an index relative to the base case scenario (income taxation):

	Std.	β=0.99	β=0.95	β=0.90	β=0.80	β=0.70	β=0.60
Production	104.94	104.94	104.93	104.92	104.90	104.88	104.85
Cons.	104.98	105.00	105.02	105.05	105.12	105.20	105.28
Cons.tax	17.58	17.57	17.53	17.49	17.39	17.28	17.17
Lab. supply	99.31	99.29	99.30	99.29	99.27	99.26	99.25
Cap. stock	123.83	123.84	123.82	123.80	123.76	123.72	123.64
Utility	106.89	106.66	106.70	106.75	106.87	107.00	107.14

Table 5. Consumption tax reform.

The endogenously computed consumption tax that yields the same revenue as the previous tax on income is 17.58% in the standard model (the row called "Cons.tax"). Notice that both for the capital stock and labor supply, the effect of the removal of taxes - ceteris paribus - is not a priori clear, since there is an income and a substitution effect. On one hand the removal of the labor income tax means that the consumer gets a higher compensation for supplying labor - but on the other hand he may choose to work less, and keep enjoy more leisure. The same argument holds for the other factor, capital: one hand the increased after-tax makes savings more rewarding, but on the other hand this means that smaller savings are required over the life cycle to reach a nest egg of a given size. To this we must add general equilibrium e¤ects, which makes the overall e¤ects unpredictable.

It turns out that the overall e¤ect on the capital stock is positive, with around 23.83% in the standard case, and slightly negative as far as the labor supply is concerned: in this case labor supply goes down with 0.69%. When the hyperbolic economies are considered the e¤ects are almost similar: with ¯=0.9 and ¯=0.7 the increase is in capital stock is 23.80% and 23.76% (respectively), and the decrease in labor supply is 0.71% and 0.74% (respectively).

The welfare e¤ect of the consumption tax is clearly positive, and in the standard Auerbach and Kotliko¤ (1987) case (¯=1) utility goes up with 6.89%. The welfare e¤ects in the hyperbolic economy is very similar: in the case where ¯=0.9 the increase is slightly smaller, 6.75%, and when ¯=0.7 the increase is slightly larger: 7.0%.

4.2 Welfare comparison

Having performed the three types of fundamental reform in the standard economy as well as in di¤erent hyperbolic economies, we can now compare the results from the simulations. The perspective is, as mentioned in the introduction, to investigate how policy experiments performed in economies with hyperbolic consumers di¤er from the standard case. For these experiments the …rst conclusion is that the e¤ects of the types of policy is quite robust across di¤erent values of ¯.

From a welfare perspective the in‡uence of ¯ was the highest in the capital income tax reform. One reason for this is the bias against savings that we saw in any of the hyperbolic benchmark economies (see table 1) - a bias that is stronger the lower the value of ¯. On top of this the capital income tax is increased from 15% to 63% (or more in some cases) - which gives an even lower capital stock in the new stationary state. Measured in welfare terms this translates into quite di¤ering welfare implications of the policy change: in the standard case welfare went down 3.44%, but even with ¯=0.95 this increased to a loss of 4.74%. And with lower levels of ¯ this is even more pronounced: for ¯=0.9 the decrease is 6.14%, and with ¯=0.7, which is the value used by Laibson et al. (2000), the decrease is as high as 12.96%. In the other scenarios with labor income taxation or consumption taxation, the welfare change from the policy change is much closer to the standard case, when comparing the welfare in the standard case to ¯=0.7. In both cases there is a higher welfare e¤ect in the hyperbolic case: a di¤erence that in the labor income experiment gives a 1.57%-points di¤erence, and in the consumption taxation experiment gives a 0.11% di¤erence.

However, it is worth noting that the (from a highest welfare criterion) preferred policy reform in all cases - independent on the speci…cation of ¯ - is the same in all cases: a change to consumption taxation. This reform gives the highest welfare increase: almost 7%. For the two other policy alternatives, the result that labor income taxation is preferred over capital income taxation, is a ranking that is independent of the size of ¯: labor income taxation is the better of the two. In other words the ranking of the three alternative scenarios are independent of the value of ¯, even though this value a¤ects the quantitative welfare results, in particular when capital income taxation is concerned.

5 Marginal excess burden

Another interesting dimension for comparing the standard and the hyperbolic model, is to use the general equilibrium model to calculate the marginal cost of funds for the various tax instruments in the model. In contrast to the simulations presented above that was of the "di¤erential incidence" kind (i.e. the revenue remained constant), the point of departure in these computations are that total revenue must increase (by say 1 percent). This allows us to compare the four tax instruments (income taxation, labor income taxation, capital income taxation and consumption taxation), and determine the instruments' e¤ects. In other words we can compare how distortionary the various tax instruments are when a higher revenue is required, which from a practical public …nance perspective conveys is important information about the interplay between the tax system and the economy.[8]

[8] The classical CGE-analysis of marginal excess burdens for the U.S. is Ballard, Shoven and Whalley (1985). For a recent analysis on the Danish DREAM model see Madsen (2000).

5.1 Methodology

As defined by Ballard et al. (1985) the (marginal) excess burden (MEB) measures "the incremental welfare costs of raising extra revenue from an already existing distortionary tax". The MEB is calculated in the following manner. First consider the benchmark equilibrium - let $\bar{u}$ denote the utility for the representative agent, and let $\bar{G}$ denote the government expenditure in the equilibrium. Next add the following constraints on the counterfactual equilibrium:

$$u = \bar{u} \tag{7}$$
$$G = (1.01)\,\bar{G} \tag{8}$$

that constrains the consumer's utility in the counterfactual to be the same as in the benchmark situation, and requires the government revenue to be 1% higher than in the benchmark situation. Notice that utility here is ex ante, i.e. for a newborn consumer - with a utility function corresponding to U_0 in equation (1). Having exogenized two variables we need to endogenize two variables. The first is a tax rate (for the tax instrument in question), that allows equation (8) to be satisfied - the other tax rates remain constant. The second variable is a compensation, -, paid to the consumer, that gives him the purchasing power to maintain the utility level $\bar{u}$ (i.e. to satisfy equation (7)). This compensation is required for the consumer's utility to remain constant under the new higher marginal taxes.

The compensation is similar to the Hicksian concept of equivalent variation. In other words asking how much would be required to compensate the consumer, such that he would be indifferent between the two regimes in question (measured in a way such that a negative amount means that he prefers the lower tax regime if no compensation is given). In the present case the consumer would have to pay higher taxes when government revenue requirements are higher, and hence we would expect him to be worse off; this translates into a negative EV. However, unlike the usual hypothetical Hicksian compensation the compensation is actually carried out; in other words it is taken into account that the compensation will influence the equilibrium outcome.

With the knowledge of the size of the compensation we can calculate the MEB in dollars as

$$MEB = \frac{-}{0.01\bar{G}} - 1$$

In the standard model (where an increased income tax generates the extra revenue) we have $G = 3.82$ and the compensation $- = 0.052$. Here the income tax needs to be increased 1.12% (from 15% to 15.17%) to get the 1% extra revenue (0.0382 dollar). This percentage clearly shows the that the extra revenue is achieved using distortionary taxation: taxes need to go up with 1.12% for revenue to increase 1%. This translates into an MEB of 0.36 dollars, which has the following interpretation: increasing revenue with 1 dollar through distortionary taxation means a loss to the consumer of 1.36 dollars. From a cost-benefit perspective this means that if the increased revenue is used for a government project, then this project should only be undertaken if the project generates a benefit to the consumer of at least 1.36 dollar: increasing an already distortionary tax is costly.

5.1.1 The compensation's timing

The procedure outlined above must be altered somewhat in the hyperbolic economies. In a deterministic standard life-cycle model the timing of the payment(s) to the consumer is unimportant; therefore it does not matter whether the compensation is transferred when the consumer enters or leaves the labor market: only the present value matters. However, with hyperbolic consumers there is a conflict between consumers at the different ages: the consumer is likely to spend the majority of the transfer when it takes place - a decision in which "later selves" might disagree. Therefore the timing of the compensation matters. In the present simulations this influence is sought minimized by paying an equal nominal amount to the consumer each year. Clearly this distributional decision will also influence the results somewhat, but is an obvious focal point for a distribution scheme.

5.2 Results

This section presents the general equilibrium MEB for the various tax types: taxation of income, labor income, capital income and consumption. Table 6 below shows the MEB for economies with various βs in the interval 1.0 - 0.6 for each of the four taxes under consideration:

Tax	Std.	β=0.99	β=0.95	β=0.90	β=0.80	β=0.70	β=0.60
Income	36.72	36.87	37.54	38.43	40.00	41.58	43.82
Lab. inc.	46.51	46.45	46.22	46.00	44.63	42.92	42.17
Cap. inc.	8.30	9.13	12.46	16.73	26.76	38.15	49.58
Consumption	2.60	2.40	3.04	3.63	4.50	5.50	6.71

Table 6. MEB for four types of tax (percent).

The MEB varies considerably over the different types of taxation: in the standard case it varies between 2.60% and 46.51%. Least costly is in this case increased consumption taxes (which in the benchmark is zero), whereas the most costly source of further revenue is increased labor income taxation.

5.2.1 Ranking the policies

A raking of the marginal excess burdens (descending after the size of the MEB) is (almost) independent of the value of β - but the numeric values of these burdens are quite unstable across the various models. For $\beta \geq 0.7$ consumption taxation is by far the least harmful source of revenue, followed by taxation of capital income. But the magnitude between the two varies considerably: where an increase in capital income is 3.2 times more costly in the standard case, it is 6.9 times more costly when β=0.7 (the Laibson et al. (2000) case). This is related to the previously mentioned increasing bias against savings for lower values of β, that make higher capital income taxes increasingly costly. The two worst sources of revenue are labor income taxation and income taxation (in which labor income taxation plays an important part).

But notice that somewhere in the interval $\beta \in]0.7; 0.6[$ the ranking changes: for $\beta = 0.7$ the most costly source of extra revenue is labor income taxation, whereas it for $\beta = 0.6$ is capital income taxation. Apart from taxation of consumption, the cheapest source of extra revenue when $\beta = 0.7$ is taxation of capital income, whereas it is labor income taxation for $\beta = 0.6$. With Laibson et al. (2000)'s interpretation of the review study by Ainslie (1992) that "most of the experimental evidence suggests that the one-year discount factor is at least 30%-40%" this switch in ranking is not good news, since it occurs exactly somewhere in the interval of values for β that "most of the experimental evidence suggests" to be the correct value.

6 Summary

This paper has examined the importance of introducing consumers with hyperbolic discounting in a standard general equilibrium model à la Auerbach and Kotlikoff (1987). This was done by performing an identical set of policy experiments on both the standard model, as well as on a number of models with hyperbolic discounting consumers - models where the "degree of hyperbolicness" varied considerably. Two types of policy experiments were performed. The first set of experiments concerned a fundamental tax reform, which in this case was a revenue neutral change in tax base - from income taxation to labor income taxation, capital income taxation or to consumption taxation. The second set of experiments computed the marginal excess burden of taxation for the four tax instruments under consideration. The question posed in the introduction was: do policy recommendations differ in economies with hyperbolic consumers when compared with the standard model? Several insights emerged from the study.

In the analyses of fundamental reform the ranking of the three alternatives was the same. In all cases was consumption taxation superior in a welfare sense, and in all cases was capital income taxation most detrimental to welfare. But whereas the gains from consumption taxation was around the same for all economies considered (hyperbolic or not), the losses from capital income taxation increased significantly the more hyperbolic the consumers. The welfare loss when switching to capital income taxation in the standard model is 3.44%, but in the hyperbolic economy with β=0.7 the loss is 12.96%.[9] The reason for the greater loss in the hyperbolic economies, is that the capital income taxation decreases the already very low incentive to save, since hyperbolic consumers (particularly those with very low values of β) tend to consume almost all their income, and save significantly less. In fact the hyperbolic property acts as a kind of distortion against savings. A large increase in the capital income taxation makes this even more pronounced, and therefore the capital stock decreases significantly, despite the general equilibrium effect from the increased interest rate.

When comparing the marginal excess burdens in the economies, the ranking was once again reasonably stable: in a welfare sense the best marginal source of revenue is consumption taxation, and the worst is labor income taxation. But for low levels of β the ranking changes, and capital income taxation

[9] This value of β was used by Laibson et al. (2000) because "most of the experimental evidence suggests that the one-year discount factor is at least 30%-40%" (p. 28).

ends up being the most costly source of extra revenue, and labor income taxation ends up as the second cheapest. Unfortunately this change in the ranking occurs exactly somewhere in the interval of values for ¯ that "most of the experimental evidence suggests" according to Laibson et al. (2000, p. 28). This means that if revenue for some reason must be lowered (for instance president Bush's proposed large tax cut), then the answer according to conventional wisdom (the standard model) would be to lower labor income taxation since it carries the highest distortion - but if consumers have hyperbolic discount functions then it would be best (in a welfare sense) to lower the capital income tax instead.[10]

To sum up, the answer to the question do hyperbolic preferences matter? is not clear - it depends on the degree of hyperbolicness (the size of ¯). For consumers with a value of ¯ that is almost unity, the policy experiments are almost not a¤ected. But for low - and according to Laibson et al. (2000) realistic - speci…cations of the degree of hyperbolicness, the answer is not so clear. The costs of capital income taxation increases drastically for low levels of ¯: both a fundamental reform towards capital income taxation is increasingly costly, and so is a marginal increase in the capital income tax rate. In fact increased capital income taxation carries, for very low levels of ¯, the highest marginal excess burden of taxation - meaning that this is the most expensive source of extra revenue. The fact that the conclusion is so sensitive to the size of ¯ suggests that further empirical research is necessary to establish whether ¯ is really in the neighborhood suggested by Laibson et al. (2000). In that case the experiments performed in this paper show that the e¤ects are important and cannot be disregarded, and accordingly this type of preferences must necessarily be incorporated into future macroeconomic modeling work.

[10] At least some consolation comes from the fact that the preferred source of extra revenue stays the same in all experiments: consumption taxation.

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