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Economic memo
26 May 2020

www.dreamgruppen.dk

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May 26, 2020

There are a number of energy markets in GreenREFORM. Energy goods present a challenge to the standard method of building CGE-models for two main reasons. The first reason is that there are typically multiple suppliers for the same homogenous energy good. A good example of this is electricity [1]. From the perspective of firms and households, electricity is a completely homogenous good. In this example, we will concentrate on two suppliers: imported and Danish-produced electricity. The agents do not care whether the electricity is produced domestically or overseas. If the price of imported electricity falls, they would - if they could choose freely - only purchase imported electricity. In the real world, there are a number of restrictions in the supply system between the two types of electricity. A fall in the price on imported electricity therefore only results in a limited substitution towards imported electricity. In a classic CGE-model, this would be described using the so-called Armington assumption, where an individual agent's demand is described using a CES-function of the two types of electricity. The elasticity of substitution in this function determines the degree of substitutability between imported and Danish-produced electricity. The substitutability reflects the structure of the electricity supply system and not the preferences of the agents nor the production technology.

The classic Armington assumption does not work as well for energy goods. This is due to the fact that energy goods are typically measured in physical units (e.g. joules). In our example, imported and domestic electricity are aggregated using the CES-function. The problem is that the inputs in the function are measured in joules, while the output is an abstract aggregated measure, which cannot be interpreted as the aggregated electricity-consumption measured in joules. We solve this problem by introducing an agent, called the *distributor*, between the supply and demand sides. In our electricity-example, the distributor represents the electricity supply system. The distributor combines the different supplies of electricity - measured in joules - to an aggregated homogenous good - measured in joules.

The second challenge to the standard modelling also relates to the fact that energy goods are typically measured in physical units, such as joules. Since we measure correct physical units, we find equivalent correct prices per physical unit. These will often differ, depending on which supplier or consumer is observed. The yearly average price

[1] In the first 20-30 years forecasted in the GreenREFORM, elecitricty production is described using a detailed supply system. This note describes the proceeding period, where the CGE-model is used to forecast electricity produciton.

on imported electricity will, for example, typically deviate from the price on Danish-produced electricity. This is mainly due to the fact that the two types of electricity are produced at different times of day and at different times during the year. Similarly, the yearly electricity prices faced by households and firms will differ.

The existence of different prices for the same homogenous good breaks with the 'law of one price', which is typically assumed in CGE-models. This is not a major problem on the supply side, where it is unproblematic to let the distributor agent face different supply prices. On the demand side, on the other hand, we face a bigger problem. If the price for individual consumers follows the average average energy price, then the combined value of supply will not follow the combined value of demand. We interpret the difference between these two as the distributor's profits. This profit is transfered to the domestic suppliers as lump sum earnings. The suppliers' marginal characteristics are therefore not affected.

Supply side

This section will describe how the distributor combines the different supplies (measured in joules) to an aggregated homogenous product (measured in joules).

Assume that there are n suppliers for the same homogenous energy good. One of these can be imports. The distributor purchases quantity c_j at price p_j from the j^{th} supplier. This gives the combined supply of the homogenous energy good:

$$C = \sum_j c_j. \tag{1}$$

We assume the existence of a CES-criteria-function, which measures the distributor's assesment of the purchased mix of goods:

$$U = \left[\sum_j \mu_j^{\frac{1}{E}} c_j^{\frac{E-1}{E}} \right]^{\frac{E}{E-1}}, E > 0$$

where U can reflect 'utility' or 'usability'. The function describes the existing substitution-options and can be said to include technological restrictions in the system, but also broader concepts, such as energy security.

The distributor wishes to acquire the homogenous energy good as cheaply as possible. For a given utility-level U, the distributor therefore minimizes the combined costs $\sum_j p_j c_j$. Since U is a CES-function, this cost-minimization results in a CES-demand-system:

$$c_j = \mu_j \left(\frac{p_j}{P^U} \right)^{-E} U, j = 1, .., n \tag{2}$$

$$P^U U = \sum_j p_j c_j \tag{3}$$

If the distributor wants a given combined supply, C, of the homogenous energy good, she must choose the utility-level U, which ensures that (1) is upheld. It can be shown that the system (1), (2) and (3) has the following solution:

$$c_j = \frac{\mu_j p_j^{-E}}{\sum_i \mu_i p_i^{-E}} C,\ j = 1,..,n \tag{4}$$

This system resembles a CES-demand-system, but has the additional characteristic, that the individual quantities c_j will always sum to the total C. Notice that the two abstract concepts U and P^U are reduced out of the problem, such that (4) only depends on the prices per joule and the energy quantities, which are measured in joules.

We can now define the average price (measured per joule) as:

$$P \equiv \frac{\sum_j p_j c_j}{C} \tag{5}$$

By inserting (4), we find that:

$$P = \sum_j \frac{\mu_j p_j^{-E}}{\sum_i \mu_i p_i^{-E}} p_j \tag{6}$$

such that P only depends on the prices, and can therefore be considered a price index.

From the perspective of the distributor, the prices, p_j, are exogenous. The average price, P, can therefore be determine from (6). The distributor is indifferent towards the size of the combined supply, C. The distributor just delivers what is demanded, which is determined in the next section. Given the combined quantity C,determined on the demand side, individual supplies, c_j, are found using (4).

Demand side

Assume that there are m consumers, whom demand the same good, but face different prices, p_i^D. One of these can be exports. An individual consumer's demand is given by:

$$c_i^D = \eta_i \left(\frac{(1+t_i)\, p_i^D}{P_i^U} \right)^{-G} U_i,\ G > 0 \tag{7}$$

where t_i is a tax, and where we can assume that U_i and P_i^U are given. This is a stylized version of a typical CES-demand in GreenREFORM.

Assume first that prices are identical. This implies that:

$$p_i^D = P,$$

where P is the average price determined in the previous section. If this was the case, c_i^D could be derived from (7), allowing us to derive:

$$C = \sum_i c_i^D. \tag{8}$$

In this way, the combined quantity C would be determined, and the model would be fully specified. This would imply that:

$$\sum_i p_i^D c_i^D = P \sum_i c_i^D = PC,$$

such that the value of the demand equals the value of the supply.

Assume instead that we, from the data, know that consumers face different prices:

$$p_i^D = (1+\xi_i)\,P,$$

where ξ_i can be both positive and negative.

We can once again derive c_i^D from (7) and C from (8). We are then faced with a problem, since it would typically hold that:

$$\sum_i p_i^D c_i^D = P\sum_i (1+\xi_i)\,c_i^D \neq PC$$

The value of the demand does not necessarily equal the value of the supply. We therefore define the distributor's profit as:

$$\pi = \sum_i p_i^D c_i^D - \sum_i p_i c_i = P\sum_i \xi_i c_i^D$$

This profit can both be positive and negative. The distributor's profit is transfered to domestic producers as lump sum earnings. This implies that the producers' first-order-conditions are unchanged, but that income circulation remains intact.

Conclusion and calibration

The supply side determines the quantities produced, c_j, and the average price, P (prices p_j are cost-based and are therefore determined in a different part of the model):

$$c_j = \frac{\mu_j p_j^{-E}}{\sum_i \mu_i p_i^{-E}} C$$

$$PC = \sum_j p_j c_j$$

The demand side determines the quantities demanded, c_i^D,the prices faced by consumers, p_i^D, the combined quantity produced, C, and the profit, π:

$$c_i^D = \eta_i \left(\frac{(1+t_i)\,p_i^D}{P_i^U}\right)^{-G} U_i$$

$$p_i^D = (1+\xi_i)\,P$$

$$\pi = P\sum_i \xi_i c_i^D$$

$$C = \sum_i c_i^D$$

When calibrating, C is the combined energy-quantity measured in joules and P is the average price per joule. We then derive:

$$\xi_i = \frac{p_i^D}{P} - 1$$

It is therefore probably reasonable to also measure p_i^D per joule.

In the baseline year, it will apply that $\pi = 0$.

The parameter μ_j can calibrated as follows:

$$\mu_j = \frac{c_j}{\sum_i c_i} p_j^E,$$

where c_j is measured in joules.