



# The Empirical Foundation of MAKRO

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## 1. Overall empirical approach

#### **MAKRO's Application Areas and Choice of Model Class**

MAKRO is a large-scale macroeconomic model, the intended use of which is described in the paper *MAKRO: Modeling Choices* alongside a non-technical introduction to MAKRO's theoretical modelling of economic agents' behavior. The present paper describes the empirical methodology chosen to appropriately reflect the properties and specific requirements pertaining to the model.

The intended use of MAKRO for medium to long-term projections, as well as impact analyses of macroeconomic policy initiatives, necessitates firstly that the model can aptly analyze the economy's convergence to underlying structural levels<sup>1</sup>. It therefore requires empirically and theoretically sound short-term properties. A prevalent assumption in modern macroeconomic literature is that of micro-founded forward-looking behavior among economic agents. This assumption is made in MAKRO, granting the opportunity to differentiate responses in behavior between temporary and permanent economic shocks (see also the discussion chapter in *MAKRO: Modeling Choices*).

Moreover, MAKRO is capable of projecting beyond short-term impact analysis as a long-term structural model. After a period of business cycle neutralization, the model converges to the empirically and theoretically substantiated structural levels. In the long term, the economy converges to the given demographic, educational and socioeconomic conditions as well as those related to, among other, technology, productivity and the foreign economy. In other words, MAKRO generates a baseline projection of the Danish economy<sup>2</sup>, setting it aside from DSGE models, which are generally defined relative to a steady-state. MAKRO is not initially in steady-state nor is it designed to be in the long-term as a consequence of a persistently evolving demography, labor force and international effects.

As the Ministry of Finance's primary tool in long-term projections and macroeconomic impact analysis, MAKRO must meticulously model public income and expenditure including taxes, transfers and public consumption. As a professional planning and budgeting tool, the model's variables must directly reflect corresponding measures in the national accounts. For instance, calculating taxes incurred by firms requires a decomposition of inputs from different sectors as well as of outputs into different final production inputs. Households' savings and consumption behavior are comprehensively modelled to allow computation of income taxes. Furthermore, demographic changes are of import to income taxes, public transfers and public consumption.

Finally, MAKRO contains a so-called overlapping generations (OLG) structure. Households are distributed by age and thus have varying distributions of consumption, income and wealth in any given year<sup>3</sup>. This added complexity is valuable in both the short- and long-run. In the short-run, responses to exogenous shocks can vary by age, more realistically reflecting heterogeneity in impulse response properties. In the long-run, overlapping generations allows for an analysis of labor market, education and pension reforms as well as a nuanced endogenous determination of households' savings, wealth and housing stock.

<sup>&</sup>lt;sup>1</sup> MAKRO will not be used as an actual forecasting model

<sup>&</sup>lt;sup>2</sup> With release of MAKRO's beta-version follows a stylised and tentative baseline projection, forming a base for impulse analysis to illustrate the model's properties. This is not intended as an actual projection for the Danish economy with existing and planned political initiatives, nor does the COVID-crisis factor into the projection.

<sup>&</sup>lt;sup>3</sup> Lifecycle profiles are based on register data and scaled to match the national accounts, see section 2.2.

As is apparent in the hitherto description of MAKRO, the model is complex and consists of many components. Individual components are to the greatest possible extent based on state-of-the-art modeling in the international macroeconomic literature. For instance, the labor market is built as search-and-matching, households are either intertemporally optimizing with rational expectations or partly liquidity constrained and firms maximize their value under quadratic installation costs.

MAKRO is largely more complex, exhaustive and non-linear than the typical DSGE model. This is made possible by a diverging simplifying assumption in MAKRO, namely that forward-looking agents have model-consistent expectations while the model itself contains no uncertainty (stochasticity) – meaning optimization occurs under perfect foresight. As concerns the absence of stochasticity, it is to be noted that MAKRO is a simpler model than its DSGE counterpart, which contains stochasticity on an aggregate level. On the one hand, this allows for a very high level of detail in the modelling of MAKRO and circumvents the need for linearization around a steady state. On the other hand, the significance of uncertainty in households' and firms' decisions will not be explicitly modelled<sup>4</sup>. This is treated by implementing specific elements in the modeling of certain areas, aligning agents' behavior with a counterfactual scenario in which uncertainty is explicitly modelled (see also: *MAKRO: Modeling Choices*). This is, for instance, risk premia for firms, debt ratios for both firms and households and an element of cautionary savings for households (financial wealth appears in the utility function).

There is likewise noteworthy uncertainty on the micro-level (which does not factor into DSGE models) affecting individual households' and firms' behavior. For example, uncertainty about lifetime income for a young person is driven more so by uncertainty about individual long-term salary and job prospects than macroeconomic business cycles. This individual uncertainty implies, as in Carroll (2001), a cautionary savings motive, which contributes to rational agents' consumption tracking their life-cycle income to some extent. Given the inability to model uncertainty on a micro-level in a large macroeconomic model, implications thereof must currently be approximated on a more aggregate level<sup>5</sup>.

#### **MAKROs empirical strategy**

MAKRO's empirical strategy is concentrated around ensuring that short-term properties of the model as a whole (especially convergence rates) and long-term characteristics (substitution elasticities) are empirically founded. As is pointed out above, the pivotal intention is for applied estimation methods to be commensurate with an exhaustive non-linear model with forward-looking behavior.

The model's short-term properties are ensured by determining its short-run or rigidity parameters so the economy's reaction to varied shocks (impulse response functions, or IRFs) in MAKRO match their empirical counterparts. This so-called IR-matching methodology is established in the DSGE literature where it is used as an alternative to (Bayesian) maximum likelihood estimation. The vast majority of the empirical IRFs are derived from SVAR models.

<sup>&</sup>lt;sup>4</sup> It should be noted that typical application of DSGE-models (see eg. The European Commission's Quest III-model, Ratto et al. 2008; The Danish National Bank's model, Pedersen 2016; The Norwegian Ministry of Finance's NORA model, Aursland et al., 2019) involve linearisation of the non-linear DSGE model. This ultimately avoids the significance of uncertainty in agents' behavior (as is the case in MAKRO), if even the stochasticity is kept.

<sup>&</sup>lt;sup>5</sup> Aspects surrounding uncertainty on the micro-level are included in heterogeneous-agent (HANK) models. This base of literature is currently in early stages as concerns application in practice. Moreover, the modelling is typically partial and bears less implication in MAKRO (see eg. Kaplan et a., 2018).

The IR analyses are carried out in practice principally around demand shocks (public consumption and foreign demand), supply shocks (labor supply) and shocks containing both elements of demand and supply (oil price and euro area interest rate). From these empirical analyses are calculated IRFs describing the economy's initial reactions as well as convergence rates for key macroeconomic variables following shocks to the system. The determination of short-run parameters to match MAKRO's short-run properties with empirical IRFs is helped by a matching algorithm.

The model's short-run properties are further verified in comparison to existing literature surrounding short-run marginal propensities to consume following temporary income shocks.

MAKRO's long-run properties are ensured by, *inter alia*, estimation of a wide variety of substitution elasticities in firms' and households' demand. To this end, the model group has developed a methodology based on the so-called Kalman filter. Through extension of a classic error correction model, this algorithm has automated an otherwise assumption-dependent analysis of the underlying structural evolution in the data. International trade elasticities are estimated with similar methodology and data to existing international trade models. In addition to the results from these estimations, the model group draws on the existing literature to determine parameters in the model.

It is noteworthy that the empirical foundation of MAKRO's short- and long-run properties stem from varied empirical approaches. This is in contrast with classic error correction models wherein long-run elasticities and convergence rates are often estimated in one concurrent (simultaneous) estimation. The view of the model group is that a varied approach for MAKRO is more appropriate for numerous reasons. Firstly, it is an established issue that the quantity and quality of data can make simultaneous estimation of long-run elasticities and convergence rates difficult. The solution is often to determine either convergence rates *or* long-run elasticities exogenously, which is not desirable in a model designed to reflect empirical foundations in the short and long-run. Secondly, as is previously noted, forward-look-ing behavior in many of MAKRO's specifications (an important element of modern macroe-conomics) makes it difficult to estimate such a system by a system of single equations.

MAKRO's ambition is to best possibly reflect the available empirical results from a broad approach. Within this is understood that results are drawn from recognized results in the literature, own estimations of certain parameters are central to the marginal behavior of agents, e.g. in face of shocks (hereafter behavioral parameters) and an empirical foundation of the overall model/economy's convergence rates following shocks to key macroeconomic variables. The empirical SVAR models, on which the latter is based, are to a great extent a-theoretical and thereby allow the data to freely express the economy's reaction to, and convergence following, macroeconomic shocks.

MAKRO is estimated neither as a collected simultaneous system (as it is too large) nor is it helped by individual estimations of single equations making up the larger system. Instead, the model is estimated through a combination of methods in a broad approach. This is both necessary and desirable given its level of detail, theoretical foundations, perfect-foresight properties as well as empirically founded convergence properties in a collected system.

#### Potential trade-offs between theory and empirics in MAKRO

The literature on macroeconomic models occasionally presents an inherent trade-off between the weights distributed between theory and empirical results (see eg. Wren-Lewis, 2019). This is in principle a false opposite: As in other scientific disciplines, economic theory must be either falsified or confirmed (or at least contradicted) by relevant and thorough empirical results. Consequently, there should not be a contradiction between sound theory and empirical results. However, in the practice of model-development, certain trade-offs can arise.

This is principally due to economic theory's simplification of a very complicated reality in order to draw parallels to empirical results. Additionally, the current base of economic theory is incomplete and uncertain in some areas.

Moreover, empirical results in economics consist of many varied approaches, each with strengths and weaknesses and none guaranteed to present a complete answer (indeed some presenting different answers). This is true for simultaneous equation estimation on macroeconomic data, SVAR models, CVAR models, Bayesian estimation of DSGE models, cross-country empirical results and models estimated on micro or disaggregated data (eg. panel data). The empirical analysis of the macroeconomy is thus also incomplete and uncertain, despite varied approaches complementing each other. The choice of an expanded theoretical foundation – eg. perfect foresight with regards to the life-cycle – excludes certain empirical approaches such as estimation of utility functions by single equations.

Pagan (2003) presents a balance between the focus on empirical foundations and theoretically strict and consistent modelling. All serious models give import to empirical results while attempting to be stringent with respect to theory. The weighting between these is however varied. DSGE models typically reflect a larger focus on theoretical consistency while traditional macroeconometric models (SEM models) primarily focus on the historical explanatory power of the models' equations. Although the theoretical foundation of MAKRO is more stringent and consistent than traditional macroeconometric models, more weight is given to empirical results than absolute theoretical consistency.

With regards to MAKRO's empirical foundation, the weighting is further distributed between the model's single behavioral equations and its overall properties. MAKRO weights the empirical foundation for its overall properties heavily – even more so than the historical explanatory power of the single behavioral equations (eg. a high R<sup>2</sup>). The latter is typically a focus in estimating single equations, on which traditional macroeconometric models are based.

A larger focus on high explanatory power of historical data pulls a model in the direction of estimating many single equations with chosen explanatory variables, where instead of derivation from explicit optimization problems, these relations are specified more freely. This approach permits a degree of risk of *ad hoc* adjustments, eg. overfitting and unstable relations. Moreover, the absence of appropriate expectations data creates a challenge (in the shape of an omitted variable), which is often "solved" through assumptions of adaptive expectations. Experiences both in Denmark and internationally have not seldom been that previously established and well-explained relations have broken down and needed adjusting, re-estimation or changing (this is a common issue with macroeconometric models). This is rarely accompanied by a concurrent and expanded foundation on which to evaluate the overall model properties. Finally, an approach without explicit optimizing behavior is not robust to the so-called Lukas Critique, ie. that behavior can change in response to a shock not documented in historical experience or structural changes, eg. policy initiatives.

Meanwhile, a large focus on theoretical consistency in the modelling of behavior, which requires simplifications in order to achieve a utilizable framework of optimizing behavior, risks explanations of historical changes which may seem constructed or reductionist. Prominent examples of this are found in the DSGE literature. These models can nonetheless be estimated as a system through (Bayesian) maximum likelihood when combined with calibration of certain parameters to representative literary estimates. This system estimation can handle forward-looking expectations – at least on a technical level – and counteract potential noise in parameter estimation from simultaneity or omitted variable bias, which is otherwise an issue in models based on single equations. System estimation is generally not adequately informative to determine parameter values in (even small) DSGE models, thus requiring supplementing information, of which the quality is important. For instance, estimations are often subject to strict limitations on parameter values (so-called *priors*), making estimation less data-driven.

There are admittedly advantages and disadvantages in the chosen approach to MAKRO's empirical foundation. The relations and parameters estimated as single equations are prone to the above-mentioned issues, an example of which being relations unstable over time. The parameters determined on the base of existing literature risk being affected by the model-developer's subjective opinions and the quality of the methods used in the relevant studies. Finally, parameters determined in the IR-matching procedure (approximately a system estimation approach) are naturally subject to uncertainty associated with estimation thereof. This uncertainty derives from, as is the case with other forms of estimation, the general variance in the data, choice and omission of variables in the SVAR, identification assumptions and the interpretation of different economic shocks along with potentially changing results following receipt of new data.

As an approach to system estimation, IR-matching is deemed to have a number of benefits as concerns transparency and interpretation. The dividing line between MAKRO and the matched empirical results (primarily SVARs) makes the choice of the latter more transparent. Furthermore, the method allows for a first visual impression of how well-matched the model is to the empirical results – as well as a decomposition into those areas well-matched and those less well-matched. Finally, the results from the SVARs and the model's potential deviations therefrom can be interpreted as originating in either model specification or estimation.

## Expectation formation is central, but in historical data, expectations are not observed

The formation of expectations in households and companies, as is well known, is central to how the economy works and how it respond to shocks. However, expectations are largely unobservable.

For traditional macro econometric models, the behavioral equations are (separately) estimated on a historical sample, and are therefore typically based on an assumption of backward-looking (static or adaptive) expectation formation. Therefore, it is relatively easy to evaluate the historical explanatory power of the estimated behavior. However, the assumption of backward-looking expectation formation can be problematic.<sup>6</sup>

A behavioral equation estimated under these assumptions with a seemingly high degree of explanatory power can be misleading, and the estimated correlation may prove to be unstable when more data becomes available. In addition, the absence of forward-looking expectations are considered to be a high price to pay when modeling an economy that is exposed to a number of expected shocks and undergoes structural changes – e.g. demographic development, indexation of retirement age, etc. Therefore, the behavioral equations in MAKRO that includes agents' expectations cannot simply be (separatelys) estimated on historical data. This is partly due to the expectations in the economy (as mentioned above) are unobservable, and partly due to the fact that it is not reasonable to assume that the expectations of forward-looking agents have historically been completely accurate. E.g. questionnairebased consumer and business confidence indicators indicated that expectations of macroeconomic developments were far too positive before the 2008 financial crisis.

<sup>&</sup>lt;sup>6</sup> The paper *MAKRO: Modeling Choices* discusses the background for forward-looking expectations in MAKRO and the distinction between forward-looking and backward-looking expectations are explained.

This also implies that evaluation of historical explanatory power of the relevant equations cannot be done in the same way as in the case of traditional macro econometric models. This can be seen as a disadvantage of the approach in MAKRO, as the historical explanatory power for a number of behavioral equations in the model are difficult to extract and assess. However, this is a necessary price to pay for the benefits of the MAKRO approach. The benefits include explicit behavioral modeling, an element of forward-looking expectations and adaptive properties that are more directly matched to empirical evidence for the economy as a whole.

#### A summary of the trade-offs in the empirical strategy of MAKRO

Overall, MAKRO is based on an explicit theoretical foundation – which is considered commonly accepted theory – but the trade-off between different empirical needs and possibilities are accepted. Thus, to some extent a less transparent historical data foundation for a series of central behavioral equations are accepted. In return, for an approach that can handle an element of forward-looking expectations and that largely is based on a direct empirical foundation of the model's adaptive properties by a series of demand and supply shocks. In addition, there is an extensive reading of the economic literature (e.g. in the determination of deep parameters and modeling of behavioral mechanisms).

As mentioned, the approach has both advantages and disadvantages in relation to the alternative (more traditional methods) and it is therefore important that the modeling choices are presented openly and justified. In addition, documentation of the most significant differences in model properties between MAKRO and selected relevant Danish macroeconomic models will be produced, so that the consequences of the chosen approach become clear.

# 2. Determination of parameters in MAKRO

MAKRO's long-term structural properties are determined - as is typically the case in CGE models - partly based on 1) economic theory, 2) estimated long-term elasticities, and 3) calibration to historical data.<sup>7</sup>

The first point is discussed in a paper about the overall modelling choices as well as the technical documentation. The current paper reviews the determination of the se second and third point. In addition, the model's short-term properties, which as mentioned are determined based on IR matching.

The determination of MAKRO's constant behavioral parameters are therefore divided into two groups: 1) parameters relating to long-term effects and which often are estimated with classical econometrics, 2) parameters relating to the short-term properties of the model, including speed of adjustment to shocks, most of these parameters are determined via a matching approach.

A vast majority of the long-term elasticities in MAKRO's production and consumption functions are estimated with a Kalman filter-based method. The method automates the inclusion of structural trends in the estimation. The MAKRO-group has developed the method for this purpose. MAKRO's foreign trade elasticities are estimated condition on product and country specific data, and afterwards aggregated. This approach - rather than estimating the foreign trade elasticities with aggregated time series data - is typically used in newer models of international trade (Hillberry & Hummels, 2013). In general, relevant results and insights from the existing empirical literature are also included in the determination of the model's elasticities, and a few parameters are only determined in this manner. Section 2.2 describes these parameters in more details.

Impulse response matching (IR matching) ensures empirically well-specified speed of adjustment in the short- and medium-term. This method is inspired by the DSGE literature (a recent example is Christiano et al, 2016), although the method used has been further developed to enable the parameterization of a large, non-linear model such as MAKRO. Section 2.3 describes the method in details. The reason why this paper primarily focus on IR matching is that the structural properties are determined in the same way or based on similar methods as CGE models (e.g. DREAM) and macroeconomic models (e.g. ADAM), while IR matching is a new method in association with estimation of parameters in large Danish macroeconomic models. The short-run properties from IR matching are evaluated in conjunction with other empirical methods and stylized facts, such as the marginal propensity to consume.

Finally, section 2.4 discusses how the empirical strategy can ensure a sufficient empirical foundation for the properties of the most central shocks that MAKRO is used to analyze.

In addition to the estimated behavioral parameters, MAKRO calibrates a large number of parameters so that the model is consistent with, among other things, the national accounts. We call this group of parameters - level parameters, as they ensure that the model variables have the correct levels where data coverage is available.

<sup>&</sup>lt;sup>7</sup> Some long-term structural properties – e.g. the labor supply's response to change in taxes and compensation rates – are for now based on calculations/estimations outside MAKRO.

The level parameters (e.g. the constant term in a CES demand function) have many similarities with so-called J-terms, known from macro econometric models, as well as measurement errors, which are often included in estimated DSGE models. In addition, the level parameters reflect that there are structural trends in the economy (e.g. effects of globalization and a growing service sector).

MAKRO uses a systematic approach in handling the statically calibrated parameters in the baseline scenario. Standard econometric methods can be used when the calibrated level parameters are treated as historical time series. This means that a vast majority of the fore-casted time dependent parameters are automated and data driven. In this way, the structural trends of the economy is captured. E.g. a higher employment growth rate in the service sector at the expense of the manufacturing sector. This is further discussed in section 2.5.

### 2.1 A general note on the determination of the parameters

As mentioned, the determination of an individual parameter in MAKRO depends on the type of parameter in question. Table 2.1 provides an overview of the empirical work and how the different types of parameters are determined. There are largely two main groups of parameters in various macroeconomic models: Behavioral and level parameters. Behavioral parameters are divided into two subgroups: The first subgroup is the separately estimated behavioral parameters, obtained from either own estimates or based on results from external empirical literature. The second subgroup is the matched behavioral parameters, determined by IR matching, see table 2.1.

#### Table 2.1 Types of parameters in MAKRO.

Parameters, main group	Parameter subset	Determination of the parameters
<b>Behavioral parameters</b> : Constant parameters which are cen-	Separately estimated behavioral parameters e.g. long-run elasticities in the production functions, privata consumption and foreign trade.	The parameters are set based on own estimations or taken from ex- ternal literature.
tral the response of agents to shocks.	<b>Matched behavioral parameters</b> (primarily short-run dynamics) e.g. in- stallation costs of capital and price rigidities.	Parameters are set through the matching of MAKRO's impulse re- sponses to their empirical counter- parts, including those from esti- mated SVAR models.
<b>Level parameters</b> : Time-varying parameters which en-	Statically calibrated level parame- ters e.g. share parameters in the in- put-output structure of the model.	Set via the calibration of the model to historical data and projected based based on historical trends (typically using ARIMA models).
sure that the endogenous variables of the model have the correct levels, based on historical data (given the behavioral parameters).	<b>Dynamically calibrated level pa-</b> <b>rameters</b> e.g. the subjective discount factor of households.	Set via calibration of the model in the final year of the historical data. Projected as a constant or based on historical trends (typically using ARIMA models).

Next, we elaborate how the long-term behavioral parameters (primarily elasticities) are estimated (section 2.2) and how the matched behavioral parameters are determined (section 2.3). Then a series of central shocks are set up. Shocks that MAKRO must be able to analyze. It is illustrated how the mechanisms and identification strategy in MAKRO contribute to the empirical foundation of the model properties through these shocks (section 2.4). Finally, it is described how the level parameters are determined through calibration and how they are forecasted (section 2.5).

### 2.2 Determination of separately estimated behavioral parameters

This section describes the determination of the separately estimated behavioral parameters and the values. These types of behavioral parameters include substitution elasticities in the model's production functions, private consumption and foreign trade. The following, discusses these three main groups in their respective subsections.

A vast majority of the behavioral parameters are estimated in-house. In addition, relevant results and insights from the external empirical literature are included in the determination of these parameters. With regard to data and method choices, the estimation follows the relevant Danish and foreign empirical literature. All the behavioral parameters discussed in this section are determined separately and not on the complete model system.

The reason way some of the estimated behavioral parameters are determined prior to the matched behavioral parameters is: First, it is assessed that these behavioral parameters are best determined using methods from the empirical literature, and there is direct focus on identifying the behavioral parameter in question. The SVAR models (which we matched to) are generally better at describing short- and medium-term adjustments and not structural or long-term behavioral reactions.<sup>8</sup>

The separately estimated behavioral parameters affect both the structural long-run and the short-run reactions. Second, it is unlikely that estimation based on a complete model system (such as IR matching) can identify all MAKRO's elasticities to a satisfactory degree. There are over 30 parameters for businesses and households alone.

For CGE models, it is normal that the elasticities are obtained from external literature or a common database, e.g. the European Commission's PREDICT 2 model (Christensen, 2015), the OECD's METRO model (OECD, 2020) or the international trade model GTAP (Hertel & van der Mensbrugghe, 2020). For DSGE models, it is also often the case that the rigidity parameters are estimated, while some of the parameters – e.g. elasticities - are taken from external literature or are calibrated conditioned on the steady state of the model, e.g. to obtain a specific long-term markup. This applies to both Bayesian estimation (Fernandez-Villaverde, 2010), and estimation based on matching to impulse responses (Christiano et al, 2016). This approach is widespread, both for medium-sized DSGE models, which are often used in the academic literature, and for larger policy-oriented models such as The Danish National Bank's DSGE model (Pedersen, 2016), the Norwegian Ministry of Finance's new model, NORA (Aursland et al, 2019) and the IMF's GIMF model (Kumhof et al, 2010).<sup>9</sup>

<sup>&</sup>lt;sup>8</sup> The method used by MAKRO to estimate the elasticities of the production functions also provides an estimate of the speed of adjustment. However, by looking at simulated data, it has been assessed that the method is primarily suitable for estimating the elasticities, since the speed of adjustment is determined with a lot more uncertainty when structural shifts occur.

<sup>&</sup>lt;sup>9</sup> There are DSGE models where the substitution elasticities are estimated with Bayesian maximum likelihood. An example is the Ramses I and II models used by the Swedish central bank. Note that the estimates are typically relative close to the mean of the chosen prior distribution (see Adolfson et al, 2007 and Adolfson et al, 2013).

In the FRBUS model - a large macroeconomic model used by the US Federal Reserve - the elasticities are obtained from external literature or calibrated with respect to the model's long-term properties, while the rigidity parameters are estimated afterwards (Brayton et al, 2014).

#### Elasticities in the production functions of MAKRO

We estimate 21 substitution elasticities in MAKRO's production functions by assuming a socalled nested CES structure.<sup>10</sup> Input in the production functions consists of machine capital, labor, building capital and materials. The elasticities are allowed to be different across the input type as well as for the 7 industries where they are estimated. As a result, the demand for factors is determined partly by the total production that increases the demand for all factors, partly by substitution between factors due to shifts in the relative factor prices, but also shifts in the efficiency of the factors (eg labor saving technological progress).

However, identification of price and technology effects cannot take place without further assumption about the technological development. As found in Antras (2004), an a priori assumption of constant, Hicks-neutral technological progress can lead to a bias in the estimates against Cobb-Douglas. In a number of recent papers, non-linear growth is allowed via a Box-Cox transformation (e.g. Klump et al, 2007), but other types of non-linear trends have also been used, e.g. on Danish data.<sup>11</sup> In MAKRO, it is assumed that technology is expressed by a sluggish and unobservable process. Thus, the problem is written up as a so-called state space model, whereby the Kalman filter can be used to estimate the elasticity at the same time as a data-driven description of potentially time-varying technological changes is obtained. This assumption is then entered into an error correction model and the long-run elasticities are estimated. A more technical description and a simulation study can be found in a separate working paper (Kronborg et al, 2019) and (Kronborg et al, 2020a).

The estimated elasticities are shown in Table 2.2. Data are on an annual frequency for the period 1967-2017 and are primarily based on national accounts figures. For the two major private sectors, manufacturing and private services, we find a substitution elasticity between machine capital and labor of 0.4-0.5. This is higher than the 0.25 that has been used in DREAM, but close to the estimates in similar estimates on Danish data (eg ADAM and Thomsen, 2015). Muck (2017) estimates the substitution elasticity between capital and labor for a number of developed countries and finds an elasticity in the range 0.3-0.7 for Denmark. Especially for building capital, it is generally difficult to find significantly positive substitution elasticities (with the exception of the extraction sector, where buildings make up a large proportion of the total factor remuneration and may have a markedly different function than in, for example, private services). One possible explanation for this may be that the data or the user-cost expression used is noisy or an imperfect measure of the true level, which will give a bias towards 0 (so-called attenuation bias). For materials, we generally find low elasticities for most sectors. As materials in MAKRO are a weighting of energy and other material inputs, this is consistent with previous estimates and other analyzes on Danish data. In general, it can be said that uncertainty about the determination of user cost contributes to a not insignificant uncertainty about the point estimates. With reference to the possible source of

<sup>&</sup>lt;sup>10</sup> The 21 estimated elasticities cover substitution elasticities between machine capital and labor, between KL aggregate and buildings and between KLB aggregate and materials for each of the 9 industries except the housing industry and the public sector, which are treated separately..

<sup>&</sup>lt;sup>11</sup> In the Danish macro model ADAM, the efficiency of the factors is also modeled non-linearly via a higher order polynomial as a function of time (ADAM, 2012). In estimations of the elasticities for the Danish Energy Agency's IntER-ACT model, Thomsen (2015) uses logistical steps to model the non-linearity of the factors' efficiency. In MONA, the production function is assumed to be Cobb-Douglas in most contexts. The same applies to SMEC and the DSGE model of the Danish central bank.

downward bias, the substitution elasticities which are estimated to be very low or to 0 are therefore set at a lower limit of 0.25 (corresponding to the lowest elasticities in DREAM).

In cases where variables are forward-looking (ie user cost for the two types of capital), we have used the relevant user cost concept in MAKRO, but typically based on static expectations.<sup>12</sup> The only exception is the real interest rate, which includes inflation expectations. To reduce the noise in the real interest rate, the expected increases in the investment price are included as a smoothed target. In practice, the expectations are unknown, so estimation of the elasticities below fully model-consistent expectations requires that it be done as part of a system estimation.

Alternatively, it could be assumed that all the shocks to the economy was known in advance, which, however, is considered to give a misleading description of the historical behavior and thus a fundamentally incorrect specification of the estimation equations. In addition, forward-looking variables introduce a significant amount of noise in price expressions, which must be expected to give a bias in the estimates towards 0 (attenuation bias).

By using a static version of MAKRO's user cost expression to estimate a long-term elasticity, we follow most of the relevant literature in the field (Chirinko, 2008). In the short term, static expectations are not model consistent, while there is no difference between static and model consistent expectations in a *steady state*. Since the elasticities are estimated in a level relation via their long-term context, it is therefore consistent to use the parameter values in MAKRO. The discrepancy that is implicit between the modeling of the expectations in MAKRO and the assumption about the expectations made in the empirical part (via the assumption about static expectations) is primarily expected to have an impact on the short-term dynamics in the estimates. In MAKRO, short-term dynamics are not determined on the basis of these separate estimates, but are largely determined by the matched behavioral parameters.

Table 2.2					
Parameters	set	based	on	own	estimations

	Note	MAKRO	ADAM	DREAM
Elasticities in the production func- tions	Kalman filter-based method.			
Materials and other (R and KLB)				
- manufacturing		0,53	(0,00)	(0,67)
- construction		0,41	(0,00)	(0,67)
- other private sectors private excl. housing		0,25*	(0,00)	(0,67)
	Energy enters the production seperat DREAM.	e from other r	materials in AD	DAM ands
Building capital and other (B and KL)				
- extraction		0,55	(0,00)	(0,25)
- other private sectors private excl. housing		0,25*	(0,00)	(0,25)

<sup>&</sup>lt;sup>12</sup> A more elegant alternative is to estimate the model with explicit expectations. For example, one could assume that agents each year had expectations similar to the Ministry of Finance ("Økonomiske redegørelse"). Collecting this data would be a big job and is not implemented due to resource constraints, but it could be an option later.

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	Note	MAKRO	ADAM	DREAM
	Energy is nestet together with with KL energi in DREAM.	. in ADAM an	d K and B are n	estet with
Machinery capital and labor (K and L)				
- manufacturing		0,51	(0,62)	(0,25)
- services		0,42	(0,42)	(0,25)
- extraction		0,33	0,00	(0,25)
- agriculture		0,25*	0,10	-
- construction		0,25*	0,41	(0,25)
- energy provision		0,25*	(0,41)	(0,25)
- sea transport		0,25*	0,00	(0,25)
	K and B are nested together with ener does not include food processing and For further details, see Kronborg et al	services are r		
Elasticities in the consumption nest	Kalman filter-based method.			
Cars and other consumption (B and EVST)		1,04	(0,69)	-
Energy and other consumption (E and VST)		0,26	(0,88)	-
Goods and other consumption (V og ST)		0,94	(1,00)	-
Services and tourism (S og T)		1,25	2,51	-
	Gasoline consumption is nestet togeth explains the lower elasticity. Further, g food is separated from other goods in borg & Kastrup (2020).	asoline is not	included in er	ergy and
Export price elasticities	Feenstra based method.			
- energivarer		5,59	-	5,00
- ikke-energivarer		5,02	2,01	5,00
- tjenester	Lig elasticiteten for ikke-energiva- rer	5,02	2,00	5,00
	Varer i ADAM er ekskl. fødevarer, skind	l, fly og biler.		
Import price elasticities				
- energy		2,56	(0,00)	1,25
- non-energy goods		2,67	0,74	5,00
- services	Set equal to the elasticity of non- energy goods.	2,67	0,58	1,25

Notes: Elasticities marked with an \* indicates, that the value has been set to a lower boundary, cf. the discussion in the text.

Source: ADAM june 2019, DREAM-code and own estimations.

#### Elasticities in the private consumption of MAKRO

The total private consumption in MAKRO is divided into 6 different consumption components (tourism, services, goods, energy, cars and housing). The division takes place by assuming a nested structure which, as for the production function, allows types of consumption to have different substitutions between them. At present, we estimate the substitution elasticities between all types of consumption, with the exception of that between housing and other consumption, which we instead take from the literature (see below).

The nested CES utility function means that the demand for the consumption types is determined partly by relative prices (the magnitude of this effect depends on the elasticities) and partly by a preference effect ("trends" in the level parameters). The preference effect expresses gradual shifts in consumer preferences over time (eg increasing demand for services). In addition to possible long-term preference shifts, the estimation equations are specified so that they can capture short-term but significant shifts in demand that cannot be explained by either shifts in prices or sluggish preference shifts. The approach chosen is very similar to that of the production elasticities and is described in (Kronborg & Kastrup, 2020b).

The estimated elasticities for private consumption are shown in Table 2.2. Data are on an annual frequency and estimated for the period 1983-2017. Services and tourism are put together at the bottom of the consumer nest and are estimated to have a substitution elasticity of 1.25. Thus, these consumption groups are substitutes as one might expect (for example, there is a similar result in ADAM, where the elasticity is significantly greater). The elasticity between goods and other consumption is estimated at 0.94, corresponding to the fact that expenditure shares are relatively constant when adjusted for a (very sluggish) trend that gives a shift from consumption of goods to services. In MAKRO, fuel consumption is included in the total energy consumption. This will tend to pull the elasticity down as the price sensitivity here is relatively small. Conversely, it is not included in car consumption, which gives a slightly higher elasticity than e.g. found in ADAM.

#### **Elasticities of foreign trade in MAKRO**

International trade in MAKRO is comprised of goods and services. Trade of goods is further separated into manufacturing and energy sectors. We have estimated import and export elasticities for the two groups of goods. The elasticities for trade of services are set equal to the estimates for non-energy goods. This choice follows the GTAP-model (Hertel & van Mensbrugghe, 2016) and other studies and is generally due to the fact that we have less detailed data related to trade of services.

Trade elasticities in the CGE-model for international trade have generally gone from being based entirely on estimates from time series to also incorporating cross-sectional data between countries as well as being estimated on a more disaggregated product-level. As discussed in Hillberry & Hummels (2013), panel data estimation helps to alleviate the potential problems, which lead to a downward bias in the time series estimates.<sup>13</sup> Imbs & Mejean (2015) constructs a multi-sector model (based on Backhus et al (1994)) based on the estimated micro-elasticities. The macro-elasticity from this model is consistent with the elasticity calculated from a weighted average of the micro-elasticities and is higher than was estimated from aggregated data.

As a result, import and export elasticities are estimated using the method in Feenstra (1994) and later Feenstra et al (2018). The method relies on using detailed trade data for many countries with detailed product groupings. This makes it possible to use heteroscedasticity of

<sup>&</sup>lt;sup>13</sup> This includes attenuation-bias, as a result of measurement errors in aggregated price indices and identification problems.

shocks in the country-dimension, to identify both the supply and demand curves. The methodological considerations are discussed in greater detail in the supplementary ocumentation (Kronborg et al (2020b)).

Table 2.2 shows the estimated elasticities for international trade in the medium-long term, which are used in MAKRO. The data is yearly from 1995-2016 and comes from the international trade database BACI. For manufacturing, we find a weighted micro-elasticity of approximately 5 in the long term. In addition, we use input-output (IO) data on a sector level to correct this estimate for consumption of domestically produced goods. This results in a value for the relevant macro import elasticities of 2.6.<sup>14</sup> This is higher than in DREAM, but in line with what is found in similar international studies. For example, Imbs & Mejean (2017) estimate the import elasticities for a number of developed countries, although not Denmark, and find an average around 5. We estimate the export elasticity to be approximately 5 in the long term, which is in line with the value in DREAM (and the Danish Central Bank's DSGE-model). We generally find that the estimated elasticities are higher when more disaggregated data is used, which is consistent with the literature.

#### Estimated behavioral parameters obtained from the literature

Table 2.3 shows an overview of the parameters in MAKRO, which are currently taken from the literature. The parameter that determines the intertemporal elasticity of substitution is set to 0.8, with reference to Havrenek (2015).<sup>15</sup> In MAKRO, utility from consumption of housing and non-housing is modelled using a so-called non-separable utility function, which means that the marginal utility from one of the consumption types depends on the consumption of the other consumption type. The majority of the literature (e.g. Khorunzhina, 2020) supports this choice. The ambition is to estimate this elasticity in the same way as for the other consumption types, but until then it is set as 0.3, which is the estimated value in ADAM (and which is used in DREAM). The elasticity from American studies on macro-data varies. For example, Piazzesi et al (2007) finds that the relationship is well-described by Cobb-Douglas preferences, while Li et al (2016) estimate the elasticity to be 0.5. American studies on micro-data typically find relatively low elasticities (for example Flavin & Nakagaw, 2008 and Stokey, 2009).

#### Table 2.3

#### Parameters taken from the literature

Parameter	Note	MAKRO	ADAM	DREAM
Intertemporal elasticity of substitu- tion	Havrenek (2015)	0.8	-	0.6
Elasticity in inheritance and utility function	Equal to intertemporal elastic- ity of substitution			

<sup>14</sup> This correction is done to account for the fact that import prices affect the domestic price level. For the same reason, other international trade models, such as GTAP, have previously used the "rule of two" as an alternative to correction using IO-data, as we have done. With this method, the substitution between domestic and foreign goods is set as half of the substitution between different foreign goods (Hillberry & Hummels, 2013). Data does not exist for own production separated by product groups. By using sector data, the implicit assumption is that own production is the same for product groups within a given sector.

<sup>15</sup> Since both wealth and inheritance are part of the household utility function, this is not the exact same intertemporal elasticity of substitution as in, for example, the CRRA utility function.

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Elasticity between housing and other consumption	ADAM (June, 2019)	0.3	0.3	0.3
Elasticity between land and housing capital	Epple et al (2010)	1.16	-	0.2
Debt-financing share of new invest- ments	VækstplanDK (FM (2013))	0.4	0.5	0.6

Notes: Since both habit as well as wealth and inheritance are part of the households' utility function, the intertemporal elasticity of substitution is not exactly the same as, for example, in a CRRA utility function.

Source: ADAM june 2019, DREAM-code and cf. note.

The elasticity of substitution between housing and land is assigned according to the existing literature, since estimation ideally requires detailed high-quality data about newly constructed buildings compared to available land. The value of 1.16 is from Epple et al (2010) and is based on American data. Similar newer studies, which include data from other countries, likewise find that the elasticity between housing and land is around one (Ahlfeldt & McMillen, 2014 and Combes et al, 2016). The chosen parameter value is higher than in DREAM<sup>16</sup> but higher than the estimate of 1.6 (although not significantly different from zero) found in Anderson (2013), where housing and land prices are modelled and estimated on an aggregated level.

#### Elasticities of the labor supply

MAKRO will, for the time being, not officially be used to evaluate labor supply effects of economic policies. With that said, it will be possible to run the model with labor supply elasticities, which are in overall agreement with the calculation methods used by the ministries. For the empirical basis, we refer to the documentation published by the ministries.

#### **Other parameters**

Table 2.4 shows a list of other parameters in MAKRO, which are not categorized in the sections above. This includes, firstly, the parameters that determine MAKRO's age profiles, secondly, the risk premiums and, finally, a few parameters that are calibrated or estimated using simple OLS-regressions.

MAKRO's OLG-structure builds on age-distributed data: Wealth, housing, income etc.. This data is based on register data (the Law Model and Statistics Denmark's wealth statistics). There is no age-distributed data for private consumption and inheritance. Private consumption is imputed as in Browning & Leth-Petersen (2003) and an age-distributed inheritance-matrix is estimated as in Boserup et al (2016). The combined system is balanced such that the totals add up to the National Accounts. The construction of age profiles is further described in Hoeck & Bonde (2021).

The second category relates to the risk premiums. A series of new, including Autrup & Hensch (2020) on Danish data and ECB (2018) for the Euro Zone, have found a tendency for firms' required rate of return to be close to constant over the last 10-15 years, despite the falling interest rate. In MAKRO, firms' required rate of return is given by the sum of the (risk

<sup>&</sup>lt;sup>16</sup> DREAM's elasticity of 0.2 reflects the generally lower estimates found in the earlier empirical literature. Newer research indicates that measurement errors in the price of land has a tendency to result in downwardly biased estimates. Studies where the price of land can be observed more precisely generally have higher estimates (see, for example, Ahlfeldt & McMille (2014) for a discussion of this).

free) interest rate and the risk premium. The historical trend is therefore that the risk premium moves opposite the interest rate. This especially appears to be the case in the period after the financial crisis, which can be explained in part by the fact that low monetary policy interest rates are the result of high uncertainty. In the baseline it is assumed that over time, lower risk premiums and higher interest rates than today will tend to cancel out, such that the required rate of returns is constant (7 pct.). This implies that higher interest rates in the future will not necessary result in significantly lower investments.

The last category of parameters are found via calibration or simple estimation. For example, the relationships between household portfolio weights (on assets) and their age as well as their total wealth are estimated through a series of OLS regressions. The separation rate for employment is calibrated using register data (Ejarque, 2021a,b).

MAKROs OLG-struktur bygger på aldersfordelt data: Formue, bolig, indkomst osv. Dette data er baseret på registerdata (Lovmodellen og Danmarks Statistiks formuestatistik). Der findes ikke data for aldersfordelt privat forbrug og arv. Privat forbrug er imputeret som i Browning & Leth-Petersen (2003) og en aldersfordelt arvematrice er estimeret som i Boserup et al (2016). Det samlede system balanceres således at totalerne stemmer med nationalregnskabet. Konstruktionen af aldersprofilerne er yderligere beskrevet i Hoeck & Bonde (2021).

#### Table 2.4 Other parameters

Parameter	Note
The subjective discount rate	Calibrated for 25-75 year olds and set equal to the long-term real inter- est rate for younger and older households.
Weight on utility from inheritance	Calibrated for 76-100 year olds and set equal to the average of these for others with probability of dying over 0.5 pct. and set to zero for younger age-groups.
Additive part of utility from wealth	Calibrated for households from 18-24 years old and set equal to the ad- ditive part of the utility from inheritance for older age-groups.
Weight on utility from wealth	Set to achieve reasonable marginal propensity to consume for the younger age-groups (scales other parameters up and down – level has limited effect on overall properties)
Risk premium for shares	Maximum of 3 percentage points and 7 percentage points minus the bond yield
Risk premium for housing	Maximum of 0 percentage points and 4 percentage points minus the bond yield. Part of the user cost of housing.
Household marginal portfolio weights	Estimated using OLS based on wealth, a constant and age-trend
Sluggishness in realization of share reevalu- tions	Estimated using OLS
Separation rate for employment	Calibrated to micro-data (Ejarque, 2021a, 2021b)

## 2.3 Determination of matched behavioral parameters

This section describes the determination of a number of parameters that are important for MAKRO's short-term dynamics. While MAKRO's long-term properties are primarily determined theoretically, with a few specifically estimated behavioral parameters, the short-term

properties and adjustment rates are to a large degree influenced by matched behavioral parameters. Examples of this include the cost of adjusting and expanding capital in production (so-called installation costs) and price rigidities. Fundamentally, the matched behavioral parameters are determined via matching of MAKRO's impulse responses to their empirical counterpart.

IR-matching is, as mentioned, a well-known method for estimating equivalent parameters in DSGE models. The method is used both in academic analysis (for example Christiano et al, 2005, 2016) and for example in the Norwegian Ministry of Finance's DSGE model, NORA (Aursland et al, 2019). In other models, where impulse responses are not directly part of the estimation, they are used to examine the model's overall properties.<sup>17</sup>

There are advantages and disadvantages with this method compared with (Bayesian) maximum likelihood estimation, which is common in DSGE models (not possible in MAKRO, c.f. section 1). Ruge-Murcia (2007) examines different methods of estimating DSGE models, including maximum likelihood and moment-based methods, where IR-matching falls into the latter. Through a number of simulation studies, it is concluded that both methods are suited for estimation of the model's structural parameters and that they give relatively accurate estimates. A possible downside with maximum likelihood estimation is that it is less robust against a miss-specified model - one that is not a precise representation of the data. On the other hand, a moment-based method will be less efficient for a well-specified model, since it uses less information. This can lead to weaker identification of the parameters (identification in MAKRO will be discussed below).

In practice, matching in MAKRO is done by choosing a number of parameters, such that MAKRO's IR-functions, for a number of central macroeconomic variables and selected shocks match the equivalent empirical impulse responses as well as possible. The parameters are chosen such that the matching is done with respects to a weighted average over the horizon of the shock. The method is similar to GMM-estimation, where the moments are the immediate reactions and adjustment rates of the different endogenous variables. The moments are therefore matched as well as possible, but it is not possible to match all moments exactly (the parameters are estimated with so-called over-identifying restrictions). The shocks are implemented far enough into the baseline that short-term economic fluctuations are assessed not to have a significant effect on the shock.<sup>18</sup> In the optimization problem, the objective function is evaluated by – simultaneously – solving the model in the case where the shock occurs and the counterfactual case, where no shock occurs. If, for example, the model is matched to three shocks, then four versions of the model must be run (including baseline) and the fit to the data is evaluated for the same parameter-vector in all cases.<sup>19</sup>

<sup>&</sup>lt;sup>17</sup> Examples of this include the FRBUS model; "Finally, after estimation the assembled model is subjected to a set of diagnostic tests to ensure that the overall system's properties are consistent with the empirical evidence, such as the dynamics of a simple VAR model." (Brayton et al, 2014) and the Danish Central Bank's DSGE model: "As revealed by the figures, some priors are set quite tight. That reflects to a large degree a necessity; without these tight priors the model would not work well in some important dimensions like impulse re-sponse functions." (Pedersen & Ravn, 2013).

<sup>&</sup>lt;sup>18</sup> In DSGE models, matching is generally done in the steady state. MAKRO – as opposed to DSGE models – does not have a clearly defined steady state. This is in part due to the fact that the model's baseline is based on a projection of population size and age-distribution and that these change over time. The baseline is therefore run for a number of years, currently until 2025, after which the shock to the model occurs.

<sup>&</sup>lt;sup>19</sup> Due to computational considerations, the entire calibration of the model is not updated, when the objective function is evaluated.

#### Advantages regarding the communication of the chosen approach

In addition to the abovementioned, purely econometric considerations, it is worth noting that IR-matching (compared with Bayesian maximum likelihood estimation) is considered to have a number of advantages with respect to transparency, interpretation and flexibility. It is relatively easy to communicate and understand that we have a series of impulse responses and that MAKRO is as consistent with these as possible. The method allows for a relatively accessible visual inspection of the degree to which MAKRO resembles the empirical model (in supplement to the value of the objective function) – and the resemblance can be evaluated in comparison with other models. Separation of the empirical model's properties and the "theoretical model's" properties also potentially allows for better evaluation of possible challenges with respect to whether matching the empirical results is due to the theoretical modelling or the empirical specification (potential problems can occur in both).

IR-matching also has the advantage, that matching can be done to different types of impulse responses (or more generally moments). These can be based on SVAR, analyses of the economy's adjustment to changes in labor supply, values for the marginal propensity to consume from the literature and more. In this way, it is relatively straightforward to combine different types of relevant empirics. This approach gives model builders a degree of choice with regards which empirical results to focus on. The explicit choice of moments used for matching makes it highly transparent, which empirical results the model builder weighs highly. These choices will be discussed further below. The method also allows others to propose and discuss alternative empirical impulse responses (or moments), that should be included in the matching.

Even if an approach based on maximum likelihood estimation had been possible, it would likely be less transparent – both for the model builder and for outside observers. In addition, the freedom that the model builder has compared using Bayesian estimation (i.e. in determination of the priors), and the consequence of this may be less transparent for people, who do not possess specialized knowledge within the field. Consideration of transparency and openness is important for the MAKRO-project, in order to ensure, as much as possible, it is realistic for external parties to participate in discussion of the empirical framework.

#### Implementation of IR-matching

In MAKRO, the basic CGE structure has been expanded with a number of mechanisms to help ensure that the model has short-term dynamics that best reflect the empirical data. Inspiration for the inclusion of these mechanisms is taken from the DSGE literature. A more thorough explanation of the choices of individual mechanisms is given in MAKRO's other documentation. Table 2.5 shows an overview of the mechanisms introduced to provide rigidity in MAKRO, as well as brief descriptions of the effects of the mechanisms.

Each mechanism has associated parameters that determine the strength of that mechanism or rigidity. Typically, these are types of rigidity that have become standard in the literature that uses DSGE models (e.g. capital installation costs). This applies both to a large part of the academic New Keynesian DSGE literature (Gali, 2009), but also to the models used in larger institutions, including the European Commission (Quest III model, see Ratto et al, 2008) and the IMF. (GIMF, see Kumhof et al, 2010). The specific motivation for the individual rigidity is stated in Table 2.5 and in connection with the other review of the model (including the description of the overall modeling choices and MAKRO's technical documentation). It is important to note that these mechanisms obviously do not alone affect the short-term dynamics of the model for the intended endogenous variable. This is partly because the mechanisms that are introduced (for example, the proportion of Hand-to-Mouth households) can

easily affect savings and investments and thus have effects in the long term, and partly because most mechanisms have general equilibrium effects. Table 2.5 therefore summarizes the primary or direct effect of the given mechanism in MAKRO. The concrete values of the key parameters regarding the short-term dynamics are described in more detail in a separate note documenting the results of the IR matching.

#### Table 2.5

#### Mechanisms affecting MAKRO's short term dynamics

Mechanism in MAKRO	Effect of mechanism
Adjustment cost on investments	Quadratic installation costs affect investments, production costs and factor composition in production via user cost.
	Contributes to the capital stock adjusting gradually and to investments re- acting gradually in the event of a shock.
Capacity utilization of capital	Capacity utilization affects output, factor composition and production costs through the production function.
Capacity utilization of labor	Provides a short-term margin of substitution between factor input and productivity. Ensures that production can increase the same year as a shock, when the productive capital stock is fixed, and that employment does not have to react more than output in the short term.
Quadratic matching costs	Quadratic matching costs are important to differentiate between the employment effect of shocks to demand and labor supply.
Export rigidities	Export price rigidity reduces the short-term impact on export demand for changes in relative prices. Export market rigidity reduces the short-term impact on demand for changes in export market growth.
Import rigidity	Reduces the short-term impact on imports of changes in relative prices.
Share of HtM-consumers	HtM consumers have a greater propensity to consume, especially in the case of temporary income shocks, and spend a larger share of housing capital gains on consumption. Ensures a sufficiently high marginal propensity to consume in the event of temporary shocks.
Housing preference of HtM-consum- ers	Gives HtM-consumers a share of homer-owned housing, increasing the reac- tion of housing investments and housing prices to income shocks.
Habits in non-housing consumption	Adds rigidity in consumption, e.g. allowing for a gradual increase in con- sumption for a permanent income shock.
Habits in housing consumption	Adds persistence to demand for housing.
Adjustment to expected housing cap- ital gains	Reduces effect of future changes in housing prices on current housing de- mand. Increases the reaction of housing prices to temporary demand shocks.
Financial frictions	Costly external finance makes firm investments more pro-cyclical as changes in earnings affects the cost of financing.
Installation costs on housing invest- ments	Adjustment costs on investments in the nest of land and housing-capital investments adds rigidity to housing investments.
Mortgage lending rigidity	Adds rigidity to the adjustment of mortgages to changes in house prices. This smoothes the short-term consumption reaction of HtM households from changes in house prices.
Wage rigidty	Wages only gradually adjust to changes in the marginal product of labor. The parameter reflects the proportion of wage contracts re-negotiated each period. Ensures that changes in labor market tightness only gradually affect wages.
Wage indexation	A share of contracts are updated based on the wages negotiated in the pre- vious period. This increases the persistence of wage changes.
Price rigidity	Prices only gradually adjust to changes in marginal costs of production. Makes inflation more persistent.

#### **Empirical impulses**

MAKRO is compared to impulse responses for temporary but persistent shocks to public expenditure, export market growth, oil prices, and interest rates, as well as a permanent shock to the labor supply, cf. also Table 2.6. In addition, other key moments are compared to other empirical results. These are, for example, results from a number of microeconometric studies (primarily on Danish data) of households' marginal propensity to consume for temporary income increases and the extent to which increased pension contributions lead to the displacement of other wealth. Table 2.7 summarizes the impulse responses and other empirical results that are central to the empirical foundation of MAKRO's short- and medium-term properties.

Shock / analysis	Variables in estimation / analysis	Comment
Shock to public expenditure	Public consumption and invest- ments, GDP, private consupmtion, private investments, unemploy- ment, GDP price deflator, wages, and housing prices.	Estimated SVAR model
Shock to export markets	Export market index, foreign prices, foreign interest rate, pri- vate consumption, private invest- ments, exports, unemployment, GDP price deflator, export prices, wages, and housing prices.	Estimated SVAR model
Shock to foreign interest rate	Same as shock to export markets.	Estimated SVAR model
Shock to oil prices	Same as shock to export markets, plus oil prices.	Estimated SVAR model
Shock to labor supply	Unemployment.	Aggregate effect from model estimated on microdata
Marginal propensity to con- sume out of temporary income	Private consumption.	Partial equilibrium version of MAKRO is compared to external empirical studies.
Marginal propensity to con- sume for shocks to housing prices	Private consumption.	Partial equilibrium version of MAKRO is compared to external empirical studies.
Displacement of other savings from mandatory pensions	Private consumption, housing in- vestments, and savings, by age.	Partial and full version of MAKRO is compared to external empirical studies.

## Table 2.6 Overview of empirical data behind MAKRO's short-term properties

Notes: All demand components, as well as the export market index, are measured as real quantities. Unemployment is analyzed based on a measure of the employment gap from Kronborg & Stephensen (2019).

Most of the empirical impulses that MAKRO tries to match are estimated using structural VAR models (SVAR). Since Sims (1980), SVARs have been one of the most widely used tools for estimating a system's endogenous response to exogenous shocks in macroeconometrics.

The effects of a shock to public expenditure are estimated, using a VAR model, on quarterly data from MONA's database for the period from 1983Q1. The identification of public expenditure shocks is based on Blanchard & Perotti (2002), who assume that fiscal policy cannot - within one quarter - respond discretionarily to other shocks to the economy, including an economic downturn. On a quarterly basis, this restriction seems reasonable given the different types of policy lags that may play a role in the decision, adoption and implementation of expenditure policy. Thus, it is assumed that the automatic effect from the business cycle on public consumption within a quarter is 0. The same identifying assumption is used in a country study by Ilzetzki et al (2013) and on Danish data by, for example, Ravn & Spange (2014) and Troelsen (2016). In addition, we control for generic domestic shocks to aggregated supply and aggregate demand (reminiscent of the approach in Mountford & Uhlig (2009)), just as foreign economic activity is assumed to not be affected by Danish shocks (the assumption of a small open economy).<sup>20</sup> The estimation of shocks to public expenditure is reviewed in a separate working paper (Kronborg, 2020b).

Foreign shocks and their effects on key domestic variables are also estimated using VAR models and quarterly data. The foreign economy is assumed to consist of a simple model with 3 variables: production / demand, prices, and interest rates. A number of ECB papers have this as a "core" in their VAR analyzes (e.g. Sousa & Zaghini (2008) and Peersman (2011)), but it is also typically the approach in the spillover literature that looks at the impact on smaller countries' economies of foreign shocks (see Jensen et al (2017) for a Danish example). In order to ensure the best possible mapping to MAKRO, it has been chosen that the specific variables consist of the foreign components included in the model, despite the fact that this provides a less theoretically rigorous model of the foreign economy itself. The alternative would have been to model the euro area and subsequently try to calculate how, for example, the euro area's prices affect the total export-competing prices (including spillovers to non-euro countries) for the Danish economy. Specifically, the rest of the world consists of an index of Denmark's export market, export-competing prices, and the money market interest rate in the euro area. The estimation of foreign shocks is reviewed in a separate working paper (Kronborg, 2020a).

The adjustment to changes in the labor supply is central to a number of adjustment mechanisms for other types of shocks in the model. While working on MAKRO, we have therefore written separate working papers that analyze this issue. The methodological considerations are described in more detail in the working paper Kronborg & Stephensen (2019). Specifically, we have established an empirical model that uses detailed administrative data at the individual level to assess what the effects of business cycle, structure, and demographics are on individuals' movements between employment and unemployment. A similar flow approach is used in much of the recent empirical literature on the labor market and macroeconomics (e.g., Shimer, 2012). In short, we do the following: First, the transition probabilities are transformed in the same way as is known from compositional data analysis (Stephensen, 2016). We use administrative data with annual frequency for the period 1980-2015. Thereafter, the structural probabilities are allowed to change gradually over time. In this way, the model can take into account factors that have significantly reduced the unemployment rate in Denmark, including a number of labor market reforms (Andersen & Svarer, 2008). Specifically, this is done by estimating a series of dynamic regressions for the transition probabilities, checking for cyclical effects and variations in demographics. Consistent with other related empirical studies, we allow for the included effects to be age-dependent (e.g., Ghosray et al, 2016). The estimated model is used, counterfactually, to analyze how unemployment would have developed if 100 new unemployed persons were exogenously added the labor

<sup>&</sup>lt;sup>20</sup> The combination of short-term restrictions and sign restrictions is implemented using the algorithm in Arias et al (2018) and originally implemented in R in a DREAM thesis (Lund-Thomsen, 2016)

force in a cyclically neutral situation.<sup>21</sup> The impulse response as well as confidence bands are calculated by performing this counterfactual experiment for the entire estimation period and for the age groups 18-50 years. As a main result, we find that the rate of adjustment (defined as the time when 95 per cent or more of the increased unemployment has disappeared) is around 4-5 years. This is roughly equivalent to the results of a similar experiment using Danmarks Nationalbank's DSGE model (Ministry of Finance, 2014). As a robustness check, we have also estimated a SVAR model based on Foroni et al (2018), where we identify a labor supply shock only by the signs of the effects on key macroeconomic variables. In this analysis, we found that the adjustment period for the unemployment rate to increased labor supply is just over 5 years, but the effect is insignificant after almost 3 years.

Usually in the literature only up to one or a few shocks are matched. This is typically the shock that is thought to be most important for the model to be able to analyze. MAKRO is matched against several estimated SVAR models at once. This puts the model to a tough test. First, the estimated responses in the different SVAR estimates will be affected by both noise and endogenous influences at a level of detail that the model does not capture. This in itself will mean that MAKRO will have a less exact fit to the individual impulse responses the more SVAR estimates are included. However, this is not in itself a problem and is to some extent an expression of the uncertainty associated with both the model and the SVAR empirics. However, problems may arise if one of the SVARs gives an impulse response which, within the framework of MAKRO, is in direct conflict with the results from the other SVAR models. Such a discrepancy may arise from the modeling framework of MAKRO, the framework of the SVAR, or the modeling of the shock. If MAKRO is the cause of the problem, the model must be corrected - otherwise, respectively the estimation of the SVAR or the modeling of the shock is adjusted.

Several times, areas of MAKRO have be developed or improved upon in order to achieve a framework that contains relevant mechanisms to match the different impulse responses. One example is quadratic labor adjustment costs, which contribute to a slower adjustment of employment in better accordance with the implicit empirical responses.

Regarding the estimation, the data itself can be a problem in the sense that there may be atypical episodes that one will not realistically be able to model. For example, developments in house prices may be characterized by a speculative increase. Given that one cannot, or it is outside the scope of the project to, set up a model that endogenously explains (specific historical) speculative house price increases, one must try to compensate in the preparation of the SVAR either by including dummies, delimiting the estimation period, or filtering data . Finally, there may be identification problems in the estimates themselves that may affect the IR functions, making them more difficult to match for a theoretical model, even if the chosen theoretical framework provides a good description of the mechanism that we attempt to be identify in historical data.

The empirical model framework can also cause problems if the included variables do not provide sufficient information on how the impulse should be translated into MAKRO. A concrete problem in relation to setting up shocks has been shocks to the foreign export markets. Here, the SVAR indicates a large increase in private consumption relative to the reaction in wages and employment. A further look at data showed that consumer confidence in Denmark had increased at the same time as the foreign export markets. With this, an underlying international optimism may have driven both the export market growth and a large part of the increase in domestic consumption. However, the consumer confidence indicator

<sup>&</sup>lt;sup>21</sup> The aggregate labor market flows from one period to the next can be calculated relatively easily via an initial condition and a matrix indicating the transition probabilities between employment, unemployment and outside the labor force.

is not explicitly included in the determination of consumption in MAKRO. We therefore match the consumption response through an ad hoc adjustment to MAKRO rather than using this response directly to identify any endogenous mechanisms. Overall, the shock may still to help identify other mechanisms.

#### Identification of parameters for short-run dynamics

From the shocks to the various estimated SVAR models, impulse responses are reported for a number of key variables (see Table 2.6). In partially estimated models, there is a 1-1 mapping between the individual variables and the parameters that belong to the considered mechanism (and which are part of a given estimated model equation). That is not the case in a system-estimation approach such as impulse-response matching. Instead, when we consider the model as a whole, parameters can be affected by any model equation where general equilibrium effects are present. The advantage is a greater focus on overall model characteristics and an estimate that can be more consistent with forward-looking expectations. The disadvantage is that there is a risk that any weakly identified mechanisms may result in the parameters potentially reflecting omitted mechanisms elsewhere in the model.

In MAKRO, the parameters controlling short-term rigidities are estimated by matching selected shocks (see Table 2.6). Regarding IR matching, it is important to be aware of whether the impulse responses (moments) matched are sufficient to ensure that the model's parameters are well identified. In order to hope for strong identification of the parameters based on the SVAR models, each of the key parameters must have a clear influence on at least one of the impulses matched to the empirical counterparts. Furthermore, the influence of the different parameters on the impulses must be sufficiently different. Overall, it is necessary that the value of each parameter has a significant effect on how MAKRO's impulses are able to match the empirical counterparts.

# 2.4 Central shocks to MAKRO: Overall empirical foundation of the model properties

Among other things, MAKRO will be used to analyze the effects of a number of different shocks to the Danish economy, which can be both temporary and permanent (and in some cases announced in advance). Table 2.7 provides an overview of the types of shocks for which it is relevant to use MAKRO to analyze. In face of these shocks, the model must provide credible and empirically based responses to a number of key macroeconomic variables. These variables include for example GDP, private consumption, exports, investment (broken down by machinery, buildings and housing), imports, domestic prices, house prices, employment, wages, and unemployment.

MAKRO is designed to include the mechanisms that are central to the effects of the relevant shocks. In this section, it is described in general terms how the chosen empirical strategy supports the empirical foundation of the relevant shocks through these mechanisms.

MAKRO's *long-term (and structural) properties* of shocks - including those mentioned in Table 2.7 - are based on the theoretical specification, the separately estimated elasticities (see section 2.2) and the calibration of the model to the data. This is basically the same approach that is also used in CGE models and macroeconometric models. The models are calibrated to the same data, and the elasticities are largely estimated from methods that are similar to each other, so the main difference lies in the theoretical specification. The formation of expectations in the model is not expected to play a decisive role for the long-term structural properties, since in the steady-state there is no difference between rational, adaptive and static expectations.<sup>22</sup>

In addition, the *short- and medium-term properties* of the model are also affected by the separately estimated elasticities. In additional to this (as described above) a number of parameters will affect the short-term reactions and medium-term adjustment in the economy, including via the extent of a number of frictions (or rigidities) in the adjustment to new structural levels. These parameters are determined by the best possible matching to the empirical impulse-response functions (cf. section 2.3).

The types of shocks where MAKRO is directly compared with empirical impulse responses regarding the short- and medium-term properties constitute a subset of the central shocks that the model must be able to be used in analyses, cf. Table 2.6 and Table 2.7. This reflects, among other things, a desire to be able to use MAKRO to analyze shocks for which it is not possible to directly estimate impulse responses for the key macroeconomic variables in a reliable manner. One of the purposes of setting up a model is precisely to use it to answer questions where there are otherwise apparent and severe empirical limitations - i.e. to obtain the best possible assessment regarding the effects of shocks on the basis of the total knowledge available. Take the VAT as an example: It has basically not changed in historically, which means that one cannot make direct empirical analyzes of previous changes in it. Thus, an assessment of the effects of change in the VAT cannot be based on such direct empirical data nor on a model where consistency with such an analysis is ensured. However, since a change in the VAT increases the price on consumption faced by households as well as decreases the real income – and given that households view these changes in the same way as other price and income changes - the assessment can be made using a model, where these types of mechanisms are empirically based, conditioned on other shocks.

It has been a point of emphasis that the chosen estimated impulse responses as well as other empirical results which MAKRO is compared to overall support the notion that the model can be used to analyze the shocks in Table 2.7. Typically, this is ensured by comparing MAKRO with empirical impulse responses for a number of the most central macroeconomic variables through several demand and supply shocks, where the various central mechanisms in the model are at play. Furthermore, the additional empirical results that the model are compared to have been selected in order to supplement the IR matching at crucial points. However, the model's assessments of some of the shocks (e.g. working time and productivity) must rely on the model's theoretical framework and empirical foundation, where the latter in these cases is to a large extent indirectly determined, e.g. with respect to the short-and medium-term adjustment. In the comment column in Table 2.7, it is briefly discussed how the effects via the most important mechanisms in the model of the various shocks are covered by the empirical approach.

It should be noted that no attempt has been made to identify differences across sectors regarding the parameters that affect companies' short-term reactions to shocks (e.g. price rigidity and installation costs on capital).<sup>23</sup> This was not considered possible in practice with any sufficient precision. Differences in the companies' reactions to shocks to the final demand components (which may be distributed differently across domestic sectors) will thus primarily be driven by, for example, differences in the input composition in the sectors, as well as any second-order effects regarding other differences between shocks.

In addition, it should be noted that the expectations formation - in contrast to what applies to the long-term or structural properties - can have a significant effect on the model's short-

<sup>&</sup>lt;sup>22</sup> Due to demographics and other factors, the model will never exactly reach a steady state, but cyclical differences will have played out.

<sup>&</sup>lt;sup>23</sup> Differences in the elasticities of their production functions have been estimated, cf. above.

term reactions and medium-term adjustment. As a consequence hereof, we should expect shocks to have different short-term effects in MAKRO, depending on whether these are very short-lived, temporary but persistent or permanent. In principle, it is very difficult empirically to isolate such expectation-driven differences in the short-term responses reliably. However, expectation-driven differences are considered to be a significant factor for the model to be able to describe (cf. also section 1), and to support this, the comparison with empirical impulse responses includes both temporary and permanent shocks. In addition, for example, the microeconometric estimates of the marginal propensity to consume compared with relate to income gains that are more short-lived than those that occur in connection with the shocks for which impulse responses are estimated.

As can be seen, the approach to the empirical foundation of the economy's short-term reactions and medium-term adjustment to the various shocks has its limitations, and it is subject to significant uncertainty. However, this is a basic condition for all models, and the task is to provide the best possible assessment of the functioning of the economy based on the available (uncertain) knowledge. In this regard, it is considered a significant advantage that the empirical foundation for MAKRO, among other things, includes direct empirical evidence for the overall properties of the economy in the determination of the central behavioral parameters.

#### Table 2.7

Stød	Kommentar		
Fiscal shocks (which may have structural elements)			
Public consumption - purchases of goods resp.	Shock to a demand component.		
employment	SVAR matching done for shock to total public consumption and investment.		
Public investment - machinery, etc. resp. buildings	Shock to a demand component.		
/ facilities	SVAR matching done for shock to total public consumption and investment.		
Transfer rates	Support for household income, which affects households' private consumption and savings. Derived structural effect on labor supply must, as a starting point, be assessed outside MAKRO. The effects of changes in household income on consumption (as well as the effects of changes in demand components in general) are at play in the shocks to which		
	SVAR is matched. In addition, the model's marginal propensity to consume is held up against microeconometric studies thereof.		
Income taxes	As for shocks to transfer rates.		
Taxes (VAT, excise duties)	Shocks affect the prices of the components of use and the real income of house- holds. Derived structural effect on labor supply must, as a starting point, be assessed outside MAKRO.		
	Effects of changes in household income are discussed above (in relation to transfer rates and income taxes). Effects of changes in the prices of the demand components are at play in the shocks to which SVAR is matched.		

Relevant shocks and empirical foundation of short-term characteristics

#### THE EMPIRICAL FOUNDATION OF MAKRO DETERMINATION OF PARAMETERS IN MAKRO

Stød	Kommentar
Corporation tax rate	Affects user cost (depending on assumptions about, among other things, the com- panies' financing behavior). Any effects through tax restructuring or relocation of the corporate tax base must be assessed outside MAKRO.
	In the long term, the effect is largely determined by the specification of user cost as well as the estimated elasticities in the production nest, while short-term effects of changes in user cost are at play in the various shocks in IR matching (including changes in foreign interest rates).
Structural policy shocks (othe	r)
Workforce - with counterpart in changing numbers on different types of transfers	Support for the total amount of efficient labor in the economy, which is converted into employment and income. Structural effect is calculated outside MAKRO.
	The adjustment of unemployment to labor supply shocks is compared with that esti- mated from e.g. aggregation of microdata. Effects of changes in household income discussed above (in relation to transfer rates and income taxes).
Structural unemployment	As for shocks to the workforce.
Average working hours	Impact on the total amount of labor in the economy.
	Effects may in the long term / structurally to a certain extent be reminiscent of other expansions of the total amount of labor in the economy, eg labor force or structural unemployment. However, short-term adaptation may differ, and its assessment must be based on the theoretical framework of the model and (in this case indirectly) empirical foundations.
Productivity growth (labor efficiency index)	Can be seen as a shock to the total amount of efficient labor in the economy (by Harrod-neutral productivity).
	Effects may in the long term / structurally to some extent be reminiscent of other expansions of the total amount of (efficient) labor in the economy, eg labor force or structural unemployment. However, short-term adaptation may differ, and its assessment must be based on the model's theoretical framework and (in this case indirectly) empirical foundation.
	Productivity shocks can also be shocks to the productivity of multiple input factors. In that case, the effects must rely on the model's theoretical framework and (in this case indirectly) empirical foundation.
The foreign economy and oth	er exogenous variables
Export market growth	Shock to a demand component. SVAR matching done for shock to foreign demand.
Oil anias	-
Oil price	Shocks to the oil price affect both Danish and foreign prices. Gives direct effects on energy prices, but also affects Danish competitiveness due to a shift in relative prices.
	SVAR matching done for shock to the oil price.
Exchange rate	Corresponds to a shock to Danish competitiveness and foreign prices, which is in- cluded in all foreign SVAR shocks.
Interest rates	Affects corporate user cost on capital, household user cost on housing, and the value and return on household wealth.
	SVAR matching done for shock to the foreign interest rate. In addition, it is the in- tention that MAKRO be held up against supplementary empirical analysis regarding interest rate effects.
Wages	The average wage is an endogenous variable in the model and the reaction depends on the source of changes in the wage.
	Wage increases due to changes in wage negotiations between employers and em- ployees affect household income as well as companies' unit costs. Effects of changes in household income discussed above (in relation to transfer rates and income taxes). Effects of changes in wages and companies' unit costs, including the impact on prices, are at play in the SVAR shocks that are matched.

# 2.5 Determination of level parameters: Calibration of historical data and projections

In MAKRO, a large number of parameters are calibrated (over 1,500 in the latest version), so that the model is consistent with, among other things, the national accounts. This type of parameter is referred to as level parameters, as their role is to ensure that MAKRO hits the right levels for the endogenous variables for which there is data-coverage. Examples of these are the distribution parameters in the model's CES functions and the depreciation rates on capital.

The vast majority of level parameters in MAKRO can be calibrated on the basis of a single relation, where the parameter is determined on the basis of the relationship between different data series. Other times, multiple parameters are calibrated at a time by solving multiple equations simultaneously. In both cases, it can be said that these series are solved by socalled *static calibration* and the level parameters can easily be determined far back in time, given the historical data. Conversely, in some cases, a level parameter is included in an equation which includes expectations for future values of variables. These level parameters are therefore determined with so-called *dynamic calibration*. In principle, one could easily construct a series for these parameters (even historically) if one was willing to use an assumption of perfect foresight (or possibly static expectations). However, this does not seem reasonable, cf. also the discussion in section 1. Therefore, this type of level parameters is only calibrated to data for the most recent data year, where the expectation of future variables is given based on the model's predictions.<sup>24</sup>

As previously mentioned, many of the level parameters of the model are projected with time series analysis so that the main structural development trends are captured. The time development in the level parameters can be assessed for the parameters determined by static calibration, which is the majority of the parameters. The time development of the level parameters over time captures several types of effects when data is viewed through MAKRO as a filter: First, as a residual in relation to the model prediction. Second, any historical structural trends such as a growing service sector. Finally, some of the parameters will include unmodeled persistence in data as well as describe structural breaks. Ideally, one wants to keep the last two effects in the projection of the model but not the former.<sup>25</sup> In this way, the long-term structural changes in the economy are maintained in the projection while a single out-lier in the base year is not allowed to affect the projection.

There is an obvious parallel between the statically calibrated level parameters in MAKRO and the so-called adjustment terms ("J-terms") in macroeconometric models (the trend in the data is captured by the estimated constant). When these models are used for projections, J-terms are often used when the residuals in the estimated equations have become systematic in recent years. In this case, one will often maintain or gradually reduce the residual rather than setting it to zero. Hendry & Clements (2003) argue that this improves the predictive power of the model if there are structural breaks in data or if models do not reflect the

<sup>&</sup>lt;sup>24</sup> A better projection of dynamically calibrated parameters could be achieved with backcasting based on the expectations the agents actually had back in time. However, this is a large project, as it requires the construction of a database of "historical expectations" for the model's most important parameters and variables, and is not part of the beta version of MAKRO.

<sup>&</sup>lt;sup>25</sup> It is likely important to include structural trends in the projection. An assumption that structural developments, which have been going on for many years, suddenly stop in the first projection year is probably a strict assumption.

true data-generating process.<sup>26</sup> Typically, it will be up to the model user whether and how the residuals are phased out in the model's projection.

In DSGE models it is possible to include measurement errors when estimating the model (e.g. Ireland, 2004), and these measurement errors can then be assumed to be white noise or allowed to have persistence. Per construction, these measurement errors are included outside the model and do not affect its dynamics, but help to correct for any dynamic mispecification (Canova, 2007). In this way, it can be said that measurement errors in DSGE models capture two of the three types of effects contained in the calibrated level parameters, namely noise in data as well as the unmodeled or misspecified dynamics found in data.

In MAKRO, we use a systematic approach to treat the statically calibrated parameters in the model projection. By considering the calibrated parameters in MAKRO as time series, one can use standard econometric methods to give a technical description of the time development of the parameters. It includes whether changes in the statically calibrated parameters are temporary or permanent, as well as the rate at which they converge toward their long-run level. Specifically, we use the procedure described in Hyndman & Khandakar (JSS, 2008) where statistical tests and information criteria are used to select the ARIMA model that best represents the data. The procedure was part of the approach used in the two best placed entries in the latest M4 forecast competition for annual data series (Fiorucci & Louzada, 2020, and Shaub, 2020).

The description of the series and the subsequent projection of the statically calibrated parameters takes place automatically and as an integrated part of running MAKRO. As part of the model output, the model user has access to a number of indicators regarding the robustness of the chosen characterization of the level parameter and has the possibility of choosing a more cautious projection of structural trends. Overall, this approach can potentially help to reduce the model user's work in assessing whether projected structural trends look "reasonable" in relation to history, thus reducing the need for hand-held corrections. However, the assumptions about the development of the parameters used in the projection will ultimately still be the model user's responsibility. Similarly, it will be the model user's responsibility to make the corrections to the technical projection that are deemed necessary, e.g. based on information or assessments outside the model.

It is also considered important to capture structural trends for dynamically calibrated parameters. However, it is not as straightforward as for the statically calibrated parameters. Ideally, dynamic calibration was done for all historical years, where the expectation terms was based on the model projection. However, this is an extremely time-consuming process which has therefore not been done yet. As a feasible alternative, we calculate the underlying trends based on static expectations. While in the short term, parameters derived by static expectations are not model-consistent, they will be in the steady state. The discrepancy (via the assumption of static expectations) is thus likely primarily related to temporary fluctuations in the parameters, while the underlying development should not be significantly affected. Thus, a reasonable assessment of the underlying trends for dynamically calibrated parameters can be obtained this way.

<sup>&</sup>lt;sup>26</sup> In MONA (2003) it is shown that the model's short-term projections have a significant bias for a number of central variables if the J-term is immediately set to zero in the basic course.

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